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EFFECTS OF POSITIONAL INFORMATION
TIME LAGS ON MOTION
STABILITY OF AUTONOMOUS VEHICLES

by

Daniel V. Venne

September, 1992

Thesis Advisor:

Fotis A. Papoulias

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**Effects Of Positional Information Time Lags
On Motion Stability Of Autonomous Vehicles**

by

**Daniel V. Venne
Lieutenant, United States Coast Guard
B.S., State University of New York, 1983**

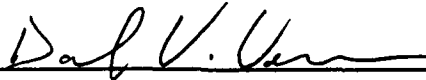
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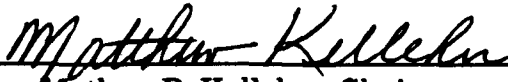


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ABSTRACT

Highly maneuverable ocean going vehicles require quick response, control, and guidance to maintain accurate track keeping characteristics. Ocean vehicles, however, may experience significant lags in their inertial positional information that limit their reaction times. This thesis investigates the effects of these lags on guidance and control. The relationship of critical visibility versus the controller time constant and its effect on the stability of the guidance/control scheme is analyzed. Results are presented based on time domain and frequency response techniques using a dynamic model of the Naval Postgraduate School Autonomous Underwater Vehicle II (NPS AUV II), for which a complete set of hydrodynamic coefficients and geometric properties is available.

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I. INTRODUCTION

A. BACKGROUND

Marine vehicles that require high maneuverability, such as hydrofoils, require quick response, control and guidance to maintain accurate track keeping characteristics. This is accomplished through successful path planning, navigation, guidance and autopilot design [Ref. 1].

Sufficient information is obtained from charted obstacles and the environment for smooth paths to be generated for the vehicle to follow [Ref. 2]. A certain level of feedback is provided through the use of sonar and acoustics in order to replan a path when uncharted obstacles are encountered or when the mission objectives are changed. Based on inertial positional information, the guidance law provides the appropriate vehicle heading by suitable use of the vehicle effectors such as rudders, dive planes, and cross body thrusters. However, lags in obtaining and processing inertial positional information can limit vehicle reaction time. In addition, the guidance law must be as fast as possible in order to ensure accurate path keeping characteristics [Ref. 3]. Therefore, the stability of the combined guidance/control scheme becomes an issue that needs to be analyzed.

B. OBJECTIVE OF THIS THESIS

This thesis investigates the effects of positional information time lags on guidance and control. The relationship of the critical visibility (minimum vehicle lookahead distance) versus the controller time constant and its effect on the stability of the combined guidance/control scheme is analyzed. Results are presented based on computations using time domain and frequency response techniques. All computations are performed for a dynamic model of the NPS AUV II [Refs. 4 and 5] for which a complete set of hydrodynamic coefficients and geometric properties is available.

C. THESIS OUTLINE

Chapter II presents the vehicle equations of motion in the horizontal plane, the equations that govern steering control, and the guidance scheme used in this research.

Chapter III presents the stability analysis based on eigenvalue computation of the combined guidance and control scheme developed in Chapter II. Results are presented based on the stability curves developed for first, second and third order approximations for time lag in the commanded vehicle heading in the control law.

Chapter IV presents an exact computation of the stability curves based on frequency response methods, which utilizes the Nyquist criterion for stability.

II. EQUATIONS OF MOTION, CONTROLLER DESIGN AND GUIDANCE SCHEME

A. INTRODUCTION

The equations of motion that describe maneuvering of a marine vehicle in the horizontal plane are presented in this chapter. A linear full state steering feedback control law is designed based on the three equations in sway, yaw, and rate of change of heading angle. The guidance scheme is based on pure pursuit guidance for path following along straight line segments.

B. EQUATIONS OF MOTION

Restricting our attention to motion in the horizontal plane (steering control), the mathematical model consists of the nonlinear sway and yaw equations of motion only. With a moving coordinate frame fixed at the vehicle's geometric center, Newton's equations of motion are

$$m(\dot{v} + ur + x_G \dot{r} - y_G r^2) = Y \quad (2.1)$$

$$I_z \dot{r} + m x_G (\dot{v} + ur) - m y_G v r = N \quad (2.2)$$

where v and r are the relative sway and yaw velocities of the moving vehicle with respect to the water; m is the mass of the vehicle; x_g , y_g are the respective lateral and longitudinal positions of the center of gravity; and Y, N represent the total excitation sway force and yaw moment, respectively. These forces can be expressed as the sum of quadratic drag terms and first order memoryless polynomials in v and r , which represent the added mass and damping due to the vehicle's motion through the water. In this way, Y and N can be represented by

$$\begin{aligned}
 Y = & \frac{\rho}{2} \ell^4 Y_{\dot{r}} \dot{r} + \frac{\rho}{2} \ell^3 (Y_{\dot{v}} \dot{v} + Y_{\dot{r}} \dot{r}) + \frac{\rho}{2} \ell^2 Y_{uv} uv \\
 & - \frac{\rho}{2} \int_{tail}^{nose} C_{D_y} h(\xi) \frac{(v + \xi r)^3}{|v + \xi r|} \xi d\xi + \frac{\rho}{2} \ell^2 Y_{\delta} u^2 \delta \\
 \\
 N = & \frac{\rho}{2} \ell^5 N_{\dot{r}} \dot{r} + \frac{\rho}{2} \ell^4 (N_{\dot{v}} \dot{v} + N_{\dot{r}} \dot{r}) + \frac{\rho}{2} \ell^3 N_{uv} uv \\
 & - \frac{\rho}{2} \int_{tail}^{nose} C_{D_y} h(\xi) \frac{(v + \xi r)^3}{|v + \xi r|} \xi d\xi + \frac{\rho}{2} \ell^3 N_{\delta} u^2 \delta
 \end{aligned}$$

where ℓ is the vehicle length, and Y_a , N_a represent partial derivatives of Y and N with respect to a , C_{D_y} is the drag coefficient, and δ is the rudder angle.

Equations (2.1) and (2.2) can be nondimensionalized with respect to the constant forward speed u , and the vehicle length ℓ ; with the dimensionless time variable being tu/ℓ .

The cross flow integral drag terms become important for hovering operations or low speed maneuvering, whereas at relatively high speeds and low angles of attack (with respect to the water), the steering response is predominantly linear.

The model becomes complete with the addition of the expressions for the vehicle yaw rate

$$\dot{\psi} = r \quad (2.3)$$

and the inertial position rate (horizontal plane)

$$\dot{y} = u \sin \psi + v \cos \psi \quad (2.4)$$

where ψ is the heading angle as shown in Figure 1.

Equations (2.1), (2.2), and (2.3) can be written as a set of three coupled linear differential equations of the form

$$\dot{\psi} = r \quad (2.5a)$$

$$\dot{v} = a_{11}uv + a_{12}ur + b_1u^2\delta \quad (2.5b)$$

$$\dot{r} = a_{12}uv + a_{22}ur + b_2u^2\delta \quad (2.5c)$$

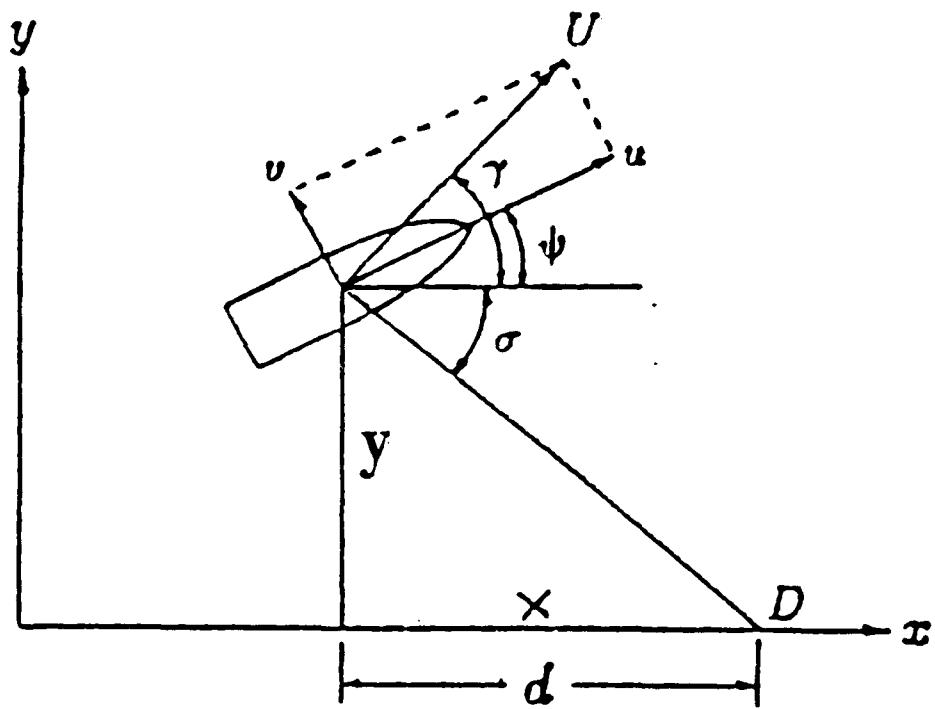


Figure 1. Vehicle Geometry and Definition of Symbols.

where the coefficients a_{ij} and b_i are functions of the vehicle hydrodynamic and geometric properties, ψ is the heading angle and δ is the rudder angle.

The linearized form of equation (2.4) at a nominal u is

$$\dot{y} = u\psi + v \quad (2.6)$$

Hence, the final set of linearized equations of motion used for steering control are

$$\psi = r \quad (2.7a)$$

$$\dot{v} = a_{11}uv + a_{12}ur + b_1u^2\delta \quad (2.7b)$$

$$\dot{r} = a_{12}uv + a_{22}ur + b_2u^2\delta \quad (2.7c)$$

$$\dot{y} = u\psi + v \quad (2.7d)$$

at any nominal u .

This system of equations becomes the basis for the controller design.

C. CONTROLLER DESIGN

Equations (2.7a), (2.7b), and (2.7c) govern the steering control of the model used in this research.

These equations can be represented in the form

$$\dot{x} = Ax + b\delta \quad (2.8)$$

where the state vector equation is

$$x = [\psi, v, r]^T \quad (2.9)$$

Linear full state feedback is introduced in the form

$$\delta = k_1(\psi - \psi_c) + k_2v + k_3r \quad (2.10)$$

where ψ_c is the commanded heading angle, and the gains k_1 , k_2 , k_3 are computed by pole placement such that the closed loop system of equations (2.8), (2.10) has the desired dynamics. The existence of the difference $(\psi - \psi_c)$ in the control law (2.9) has the effect of trying to point the vehicle's longitudinal axis towards the desired heading.

If the desired characteristic equation is selected so that its time constant is t_c dimensionless seconds, its general form becomes

$$\lambda^3 + \alpha_1 \lambda^2 + \alpha_2 \lambda + \alpha_3 = 0 \quad (2.11)$$

where

$$\alpha_1 = \frac{3}{t_c}, \quad \alpha_2 = \frac{3}{t_c^2}, \quad \alpha_3 = \frac{1}{t_c^3}$$

The controller gains are computed from

$$k_1 = \frac{\alpha_3}{(b_2 a_{11} - b_1 a_{21}) u^3} \quad (2.12a)$$

$$(b_1 a_{22} - b_2 a_{12}) u^3 k_2 + (b_2 a_{11} - b_1 a_{21}) u^3 k_3 = \alpha_2 + b_2 u^2 k_1 \quad (2.12b)$$

$$b_1 u^2 k_2 + b_2 u^2 k_3 = -\alpha_1 - (a_{11} + a_{22}) u \quad (2.12c)$$

Due to the explicit dependence on u , the gains k_1 , k_2 , k_3 are continuously updated for different nominal forward speeds.

D. GUIDANCE SCHEME

The guidance scheme used in this research is an orientation based command scheme. In this scheme, the

commanded vehicle heading angle ψ_c is a function of the line of sight angle σ between the actual vehicle position and a reference position on the nominal path which is located at a constant lookahead or visibility distance d ahead of the vehicle as shown in Figure 1.

This line of sight angle is defined by

$$\tan\sigma = -\frac{y}{d} \quad (2.13)$$

The autopilot is then called upon to deliver the commanded heading angle ψ_c . The simplest orientation guidance law is pure pursuit guidance where the commanded heading angle ψ_c in the control law (2.9) equals the line of sight angle σ in (2.13).

Then ψ_c is defined by

$$\psi_c = -\arctan\frac{y}{d} \quad (2.14)$$

For relatively small angles

$$\psi_c = -\arctan\frac{y}{d} \approx -\frac{y}{d} \quad (2.15)$$

This results in a three state heading autopilot design which can exhibit very robust characteristics. However, the stability of this scheme may become an issue when transiting along straight line segments since the commanded heading angle is a function of the vehicle response, i.e., autopilot response is limited by time lags in vehicle dynamics. We will next examine the stability of this scheme.

III. EIGENVALUE ANALYSIS

A. INTRODUCTION

In this chapter, we present the stability analysis, based on eigenvalue computation, of the combined guidance and control scheme developed in Chapter II. A brief presentation of the stability curve is developed first for the case of zero time lag. Incorporation of a nonzero time lag in the positional information formally results in an infinite state space system. This is truncated into first, second, and third order approximations. The resulting eigenvalue problems are then analyzed with emphasis on stability curves.

B. STABILITY CONSIDERATIONS AND TIME LAG

1. Stability Considerations

In pure pursuit guidance, where the commanded heading angle ψ_c in the control law equals the line of sight angle σ , the trivial equilibrium state which corresponds to straight line motion is characterized by

$$\psi=v=r=y=0 \quad (3.1)$$

Linearization of the state equations in the vicinity of (3.1)

produces the linear system

$$\dot{x} = Ax, \quad (3.2)$$

where the complete state vector is

$$x = [\psi, v, r, y]^T \quad (3.3)$$

Local stability properties of (3.1) are then established by the eigenvalues of A . The characteristic equation of (3.2) is a quartic of the form

$$\lambda^4 + B\lambda^3 + C\lambda^2 + D\lambda + E = 0, \quad (3.4)$$

where the coefficients B, C, D, E are functions of the vehicle properties, the lookahead distance d , and the controller gains k_1, k_2, k_3 . A pair of complex conjugate roots of (3.4) crosses the imaginary axis when the following conditions are met [Ref. 6].

$$BCD - B^2E - D^2 = 0, \quad (3.5a)$$

$$\frac{D}{B} > 0 \quad (3.5b)$$

These two conditions (3.5a) and (3.5b) translate to

$$a_1 d^2 + a_2 d + a_3 = 0, \quad (3.6a)$$

$$d > \frac{b_1 a_{22} - b_2 a_{12} - b_2}{b_2 a_{11} - b_1 a_{21}}, \quad (3.6b)$$

where

$$a_1 = \alpha_1 \alpha_2 - \alpha_3 \quad (3.7a)$$

$$a_2 = \frac{(\alpha_1 \alpha_2 - 2\alpha_3)(b_1 a_{22} - b_2 a_{12} - b_2)}{b_2 a_{11} - b_1 a_{21}} - \frac{b_1 \alpha_1 \alpha_3}{(b_2 a_{11} - b_1 a_{21}) u} - \alpha_1^2 u \quad (3.7b)$$

$$a_3 = \frac{-(b_1 a_{22} - b_2 a_{12} - b_2) [b_1 \alpha_1 + (b_1 a_{22} - b_2 a_{12} - b_2) u] \alpha_3}{(b_2 a_{11} - b_1 a_{21})^2 u} \quad (3.7c)$$

The conditions (3.5a) and (3.5b) determine the critical visibility d_{crit} for stability. For $d > d_{crit}$, the equilibrium state (3.1) is stable which means that the control law will drive and keep the vehicle onto the straight line path. For $d < d_{crit}$, the equilibrium state loses its stability and the vehicle response becomes oscillatory as a result of the pair

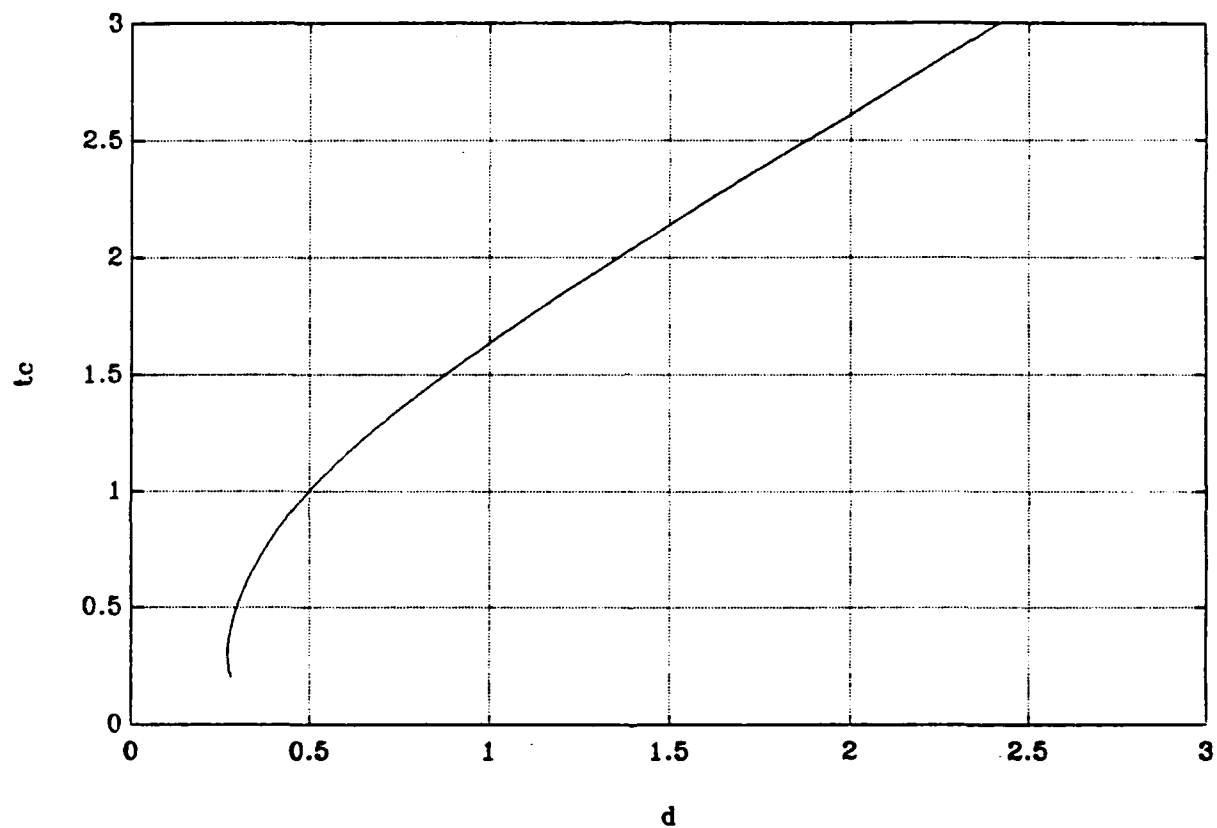
of complex conjugate eigenvalues with positive real parts.

Results of the critical visibility versus the controller time constant t_c for zero time lag are presented in Figure 2. As expected, the higher values of t_c (i.e., softer controller) require higher lookahead distances d for path accuracy. This agrees with existing guidelines that the guidance law must be sufficiently slower than the controller in order for the dynamics of the two not to interfere with each other and, therefore, guarantee stability of the combined scheme. Very high values of d correspond to a very slow guidance law with a loss in speed of response and path accuracy [Ref. 1].

2. Time Lag

In ocean vehicles, all states needed in the control law are readily available at the required rate, with the possible exception of the positional information y (Figure 1). The latter may require a significant data analysis and reduction of sonar returns and inertial navigational information. As a result, it is likely that a time lag will exist in the positional information y which is used in the guidance law.

With the introduction of a time lag T (sec), the commanded rudder angle ψ_c in the pursuit guidance control law becomes



**Figure 2. Critical Visibility Distance d versus t_c for
Zero Time Lag T .**

$$\psi_c = -\arctan \frac{y(t-T)}{d} \quad (3.8)$$

and for relatively small angles

$$\psi_c \approx -\frac{y(t-T)}{d} \quad (3.9)$$

The resultant rudder control law (2.9) becomes

$$\delta = k_1 \psi + k_1 \frac{y(t-T)}{d} + k_2 v + k_3 r \quad (3.10)$$

After some algebra, the resultant linearized equations of motion (2.7a), (2.7b), (2.7c), (2.7d) become

$$\dot{\psi} = r \quad (3.11a)$$

$$\dot{v} = b_1 u^2 k_1 \psi + (a_{11} u + b_1 u^2 k_2) v + (a_{12} u + b_1 u^2 k_3) r + b_1 u^2 \frac{k_1}{d} y(t-T) \quad (3.11b)$$

$$\dot{r} = b_2 u^2 k_1 \psi + (a_{21} u + b_2 u^2 k_2) v + (a_{22} u + b_2 u^2 k_3) r + b_2 u^2 \frac{k_1}{d} y(t-T) \quad (3.11c)$$

$$\dot{y} = u\psi + v \quad (3.11d)$$

The Taylor expansion for the term $y(t-T)$ is

$$y(t-T) \approx y - T\dot{y} + \frac{T^2}{2}\ddot{y} - \frac{T^3}{6}\ddot{\ddot{y}} + \dots \quad (3.12)$$

We can now examine the stability of the previously developed control scheme for first, second and third order approximations for time lag in the control law.

C. FIRST ORDER APPROXIMATION FOR TIME LAG

For a first order approximation for time lag T

$$y(t-T) \approx y - T\dot{y} \quad (3.13)$$

where

$$\dot{y} = u\psi + \dot{v} \quad (3.14)$$

After some algebra, the resultant linearized equations of motion (3.11a), (3.11b), (3.11c), (3.11d) become

$$\psi = r \quad (3.15a)$$

$$\dot{v} = (b_1 u^2 k_1 - b_1 u^3 \frac{k_1}{d} T) \psi + (a_{11} u + b_1 u^2 k_2 - b_1 u^2 \frac{k_1}{d} T) v + (a_{12} u + b_1 u^2 k_3) r + b_1 u^2 \frac{k_1}{d} y \quad (3.15b)$$

$$\dot{r} = (b_2 u^2 k_1 - b_2 u^3 \frac{k_1}{d} T) \psi + (a_{21} u + b_2 u^2 k_2 - b_2 u^2 \frac{k_1}{d} T) v + (a_{22} u + b_2 u^2 k_3) r + b_2 u^2 \frac{k_1}{d} y \quad (3.15c)$$

$$\dot{y} = u\psi + v \quad (3.15d)$$

The state space form of the equations of motion is

$$\dot{x} = Ax \quad (3.16)$$

In matrix form, (3.15a), (3.15b), (3.15c), (3.15d) become

$$\begin{pmatrix} \dot{\psi} \\ \dot{v} \\ \dot{r} \\ \dot{y} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ (b_1 u^2 k_1 - b_1 u^3 \frac{k_1}{d} T) & (a_{11} u + b_1 u^2 k_2 - b_1 u^2 \frac{k_1}{d} T) & (a_{12} u + b_1 u^2 k_3) & (b_1 u^2 \frac{k_1}{d}) \\ (b_2 u^2 k_1 - b_2 u^3 \frac{k_1}{d} T) & (a_{21} u + b_2 u^2 k_2 - b_2 u^2 \frac{k_1}{d} T) & (a_{22} u + b_2 u^2 k_3) & (b_2 u^2 \frac{k_1}{d}) \\ u & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} \psi \\ v \\ r \\ y \end{pmatrix} \quad (3.17)$$

The standard eigenvalue problem (3.17) is solved to determine numerically the critical d versus t_c curve for a given value of T . Appendix A presents the program used for

the first order approximation for time lag.

Figure 3 shows the resulting stability curves for the first order case. It shows the relation between the critical visibility d and the controller time constant t_c for values of time lag of 0.0, 0.5, 1.0, 1.5 and 2.0 seconds.

As expected, the stability curve shifts to the right with increasing values of time lag. For the same time constant t_c , the critical visibility distance increases with corresponding increases in the amount of time lag.

D. SECOND ORDER APPROXIMATION FOR TIME LAG

For a second order approximation for time lag T , we have

$$y(t-T) \doteq y - T\dot{y} + \frac{T^2}{2}\ddot{y} \quad (3.18)$$

where

$$\dot{y} = u\psi + v, \quad \ddot{y} = ur + \dot{v} \quad (3.14), (3.19)$$

Following some algebra, the resultant linearized equations of motion (3.11a), (3.11b), (3.11c), (3.11d) become

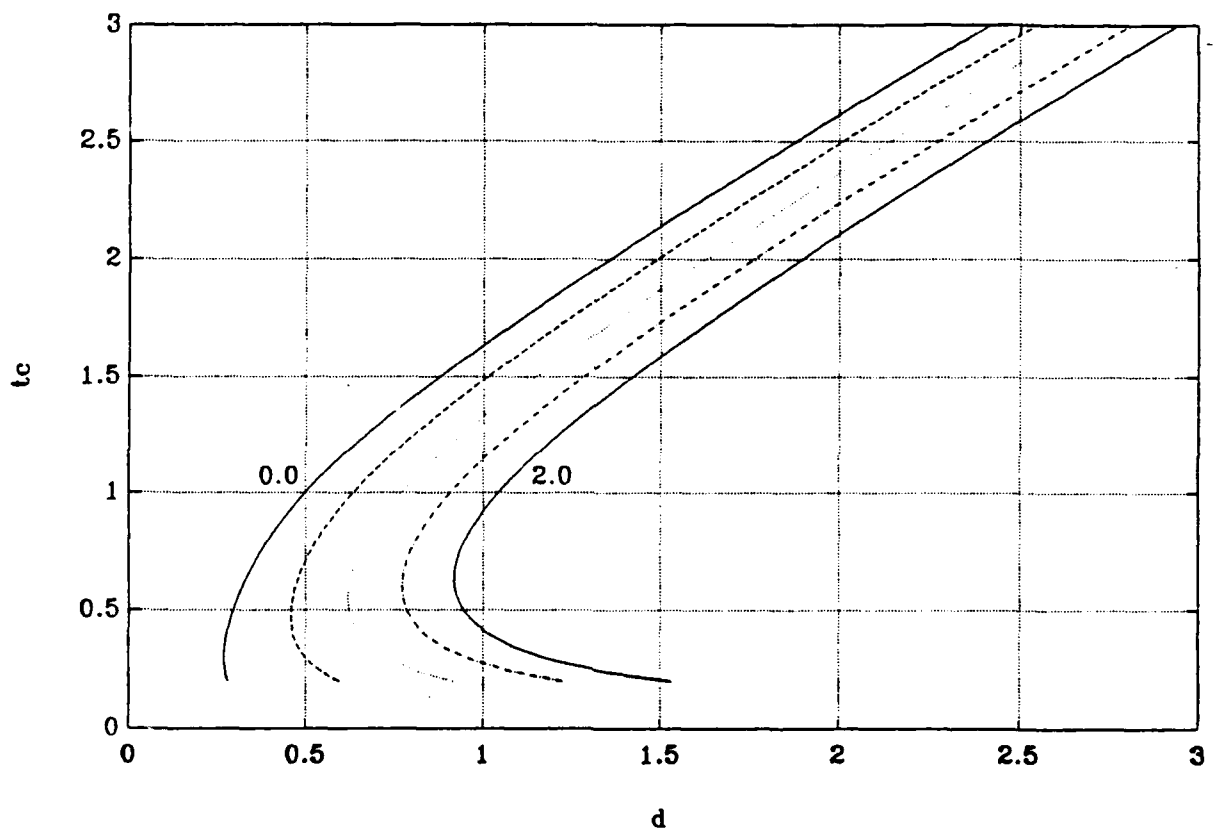


Figure 3. First Order Approximation For Time Lag:
Critical Visibility Distance d versus t_c for
Time Lags $T = 0.0, 0.5, 1.0, 1.5, 2.0$ sec.

$$\psi = r \quad (3.20a)$$

$$\dot{v} = A_{21}\psi + A_{22}v + A_{23}r + A_{24}y \quad (3.20b)$$

$$\dot{r} = A_{31}\psi + A_{32}v + A_{33}r + A_{34}y \quad (3.20c)$$

$$\dot{y} = u\psi + v \quad (3.20d)$$

where

$$A_{21} = \frac{b_1 u^2 k_1 - b_1 u^3 \frac{k_1}{d} T}{1 - b_1 u^2 \frac{k_1}{2d} T^2} ,$$

$$A_{22} = \frac{a_{11}u + b_1 u^2 k_2 - b_1 u^2 \frac{k_1}{d} T}{1 - b_1 u^2 \frac{k_1}{2d} T^2} ,$$

$$A_{23} = \frac{a_{12}u + b_1 u^2 k_3 + b_1 u^3 \frac{k_1}{2d} T^2}{1 - b_1 u^2 \frac{k_1}{2d} T^2} ,$$

$$A_{24} = \frac{b_1 u^2 \frac{k_1}{d}}{1 - b_1 u^2 \frac{k_1}{2d} T^2} ,$$

$$A_{31} = (b_2 u^2 k_1 - b_2 u^3 \frac{k_1}{d} T) + (b_2 u^2 \frac{k_1}{2d} T^2) \frac{(b_1 u^2 k_1 - b_1 u^3 \frac{k_1}{d} T)}{(1 - b_1 u^2 \frac{k_1}{2d} T^2)} ,$$

$$A_{32} = (a_{21} u + b_2 u^2 k_2 - b_2 u^2 \frac{k_1}{d} T) + (b_2 u^2 \frac{k_1}{2d} T^2) \frac{(a_{11} u + b_1 u^2 k_2 - b_1 u^2 \frac{k_1}{d} T)}{(1 - b_1 u^2 \frac{k_1}{2d} T^2)} ,$$

$$A_{33} = (a_{22} u + b_2 u^2 k_3 + b_2 u^3 \frac{k_1}{2d} T^2) + (b_2 u^2 \frac{k_1}{2d} T^2) \frac{(a_{12} u + b_1 u^2 k_3 + b_1 u^3 \frac{k_1}{2d} T^2)}{(1 - b_1 u^2 \frac{k_1}{2d} T^2)}$$

$$A_{34} = (b_2 u^2 \frac{k_1}{d}) + (b_2 u^2 \frac{k_1}{2d} T^2) \frac{(b_1 u^2 \frac{k_1}{d})}{(1 - b_1 u^2 \frac{k_1}{2d} T^2)}$$

As with the first order case, the state space equations of motion reduce to

$$\dot{x} = Ax \quad (3.16)$$

In matrix form, (3.20a), (3.20b), (3.20c), (3.20d) become

$$\begin{bmatrix} \dot{\psi} \\ \dot{v} \\ \dot{r} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ A_{21} & A_{22} & A_{23} & A_{24} \\ A_{31} & A_{32} & A_{33} & A_{34} \\ u & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \psi \\ v \\ r \\ y \end{bmatrix} \quad (3.21)$$

Similar to the first order case, the standard eigenvalue problem (3.21) is solved using Fortran programming and MATLAB in order to develop the stability characteristics of the second order case. Appendix B presents the program used to develop the stability characteristics for this case.

Figure 4 shows the resulting stability curves for this case. The results of the critical visibility d versus the controller time constant t_c for time lags of 0.0, 0.5, 1.0, 1.5 and 2.0 secs are shown.

As with the first order case, the higher values of t_c require higher lookahead distances d for path accuracy. High values of d correspond to a slow guidance law with a loss in speed of response and path accuracy, and the stability curve shifts to the right with increasing values of time lag.

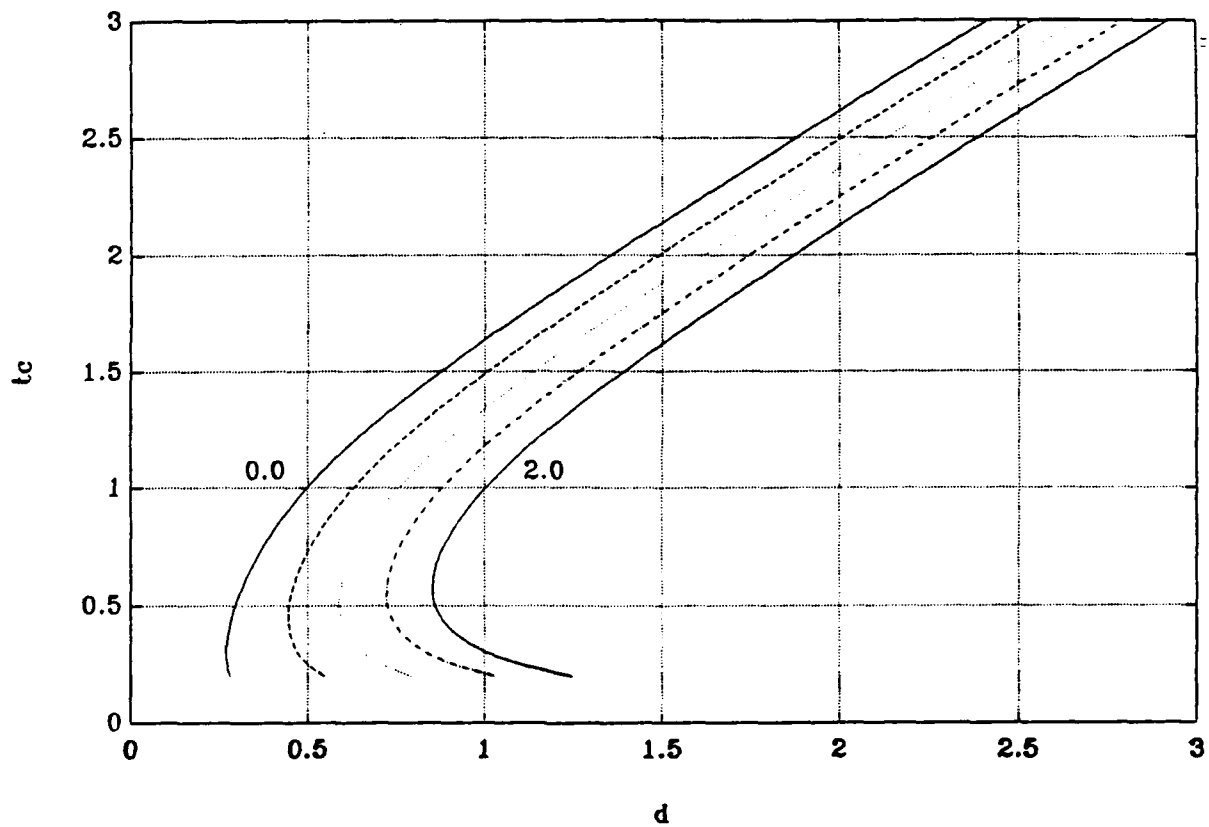


Figure 4. Second Order Approximation For Time Lag:
Critical Visibility Distance d versus t_c for
Time Lags $T = 0.0, 0.5, 1.0, 1.5, 2.0$ sec.

However, due to being more accurate than the first order case, the second order approximation provides tighter stability curves for the same time lags.

E. THIRD ORDER APPROXIMATION FOR TIME LAG

For a third order approximation for time lag T

$$y(t-T) \doteq y - T\dot{y} + \frac{T^2}{2}\ddot{y} - \frac{T^3}{6}\ddot{\tilde{y}} \quad (3.22)$$

where

$$\dot{y} = u\psi + v, \quad \dot{\tilde{y}} = ur + \dot{v}, \quad \ddot{\tilde{y}} = u\dot{r} + \ddot{v} \quad (3.14), (3.19), (3.23)$$

After a considerable amount of algebra and following a similar procedure as with both the previous two cases, the resultant linearized equations of motion reduce to the state space form

$$B\dot{x} = Ax \quad (3.24)$$

or

$$\dot{x} = B^{-1}Ax$$

The resultant linearized equations of motion (3.11a), (3.11b), (3.11c), (3.11d) become

$$\psi = r \quad (3.25a)$$

$$B_{33}\dot{r} + B_{35}\dot{v}_2 = A_{31}\psi + A_{32}v_1 + A_{33}v_2 + A_{34}r + A_{35}y \quad (3.25b)$$

$$B_{53}\dot{r} + B_{55}\dot{v}_2 = A_{51}\psi + A_{52}v_1 + A_{53}v_2 + A_{54}r + A_{55}y \quad (3.25c)$$

$$\dot{y} = u\psi + v_1 \quad (3.25d)$$

$$\dot{v}_1 = v_2 \quad (3.25e)$$

where

$$B_{33} = b_1 u^3 \frac{k_1}{6d} T^3$$

$$B_{35} = b_1 u^2 \frac{k_1}{6d} T^3$$

$$A_{31} = b_1 u^2 k_1 - b_1 u^3 \frac{k_1}{d} T$$

$$A_{32} = a_{11}u + b_1u^2k_2 - b_1u^2\frac{k_1}{d}T$$

$$A_{33} = a_{12}u + b_1u^2k_3 + b_1u^3\frac{k_1}{2d}T^2$$

$$A_{34} = b_1u^2\frac{k_1}{d}$$

$$A_{35} = b_1u^2\frac{k_1}{2d}T^2 - 1$$

and

$$B_{53} = 1 + b_2u^3\frac{k_1}{6d}T^3$$

$$B_{55} = b_2u^2\frac{k_1}{6d}T^3$$

$$A_{51} = b_2u^2 - b_2u^3\frac{k_1}{d}T$$

$$A_{52} = a_{21}u + b_2u^2k_2 - b_2u^2\frac{k_1}{d}T$$

$$A_{53} = a_{22}u + b_2u^2k_3 + b_2u^3 \frac{k_1}{2d}T^2$$

$$A_{54} = b_2u^2 \frac{k_1}{d}$$

$$A_{55} = b_2u^2 \frac{k_1}{2d}T^2$$

In matrix form, (3.25a), (3.25b), (3.25c), (3.25d), (3.25e) become

$$\begin{vmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & B_{33} & 0 & B_{35} \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & B_{53} & 0 & B_{55} \end{vmatrix} \begin{vmatrix} \Psi \\ \dot{v}_1 \\ r \\ \dot{y} \\ \dot{v}_2 \end{vmatrix} = \begin{vmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ A_{31} & A_{32} & A_{33} & A_{34} & A_{35} \\ u & 1 & 0 & 0 & 0 \\ A_{51} & A_{52} & A_{53} & A_{54} & A_{55} \end{vmatrix} \begin{vmatrix} \Psi \\ v_1 \\ r \\ y \\ v_2 \end{vmatrix} \quad (3.26)$$

Similar to both the first order and second order cases, the generalized eigenvalue problem (3.21) is solved using Fortran programming and MATLAB in order to develop the stability characteristics for the third order case. Appendix C presents the program used to develop the stability characteristics for this case.

Figure 5 shows the resulting stability curves for this

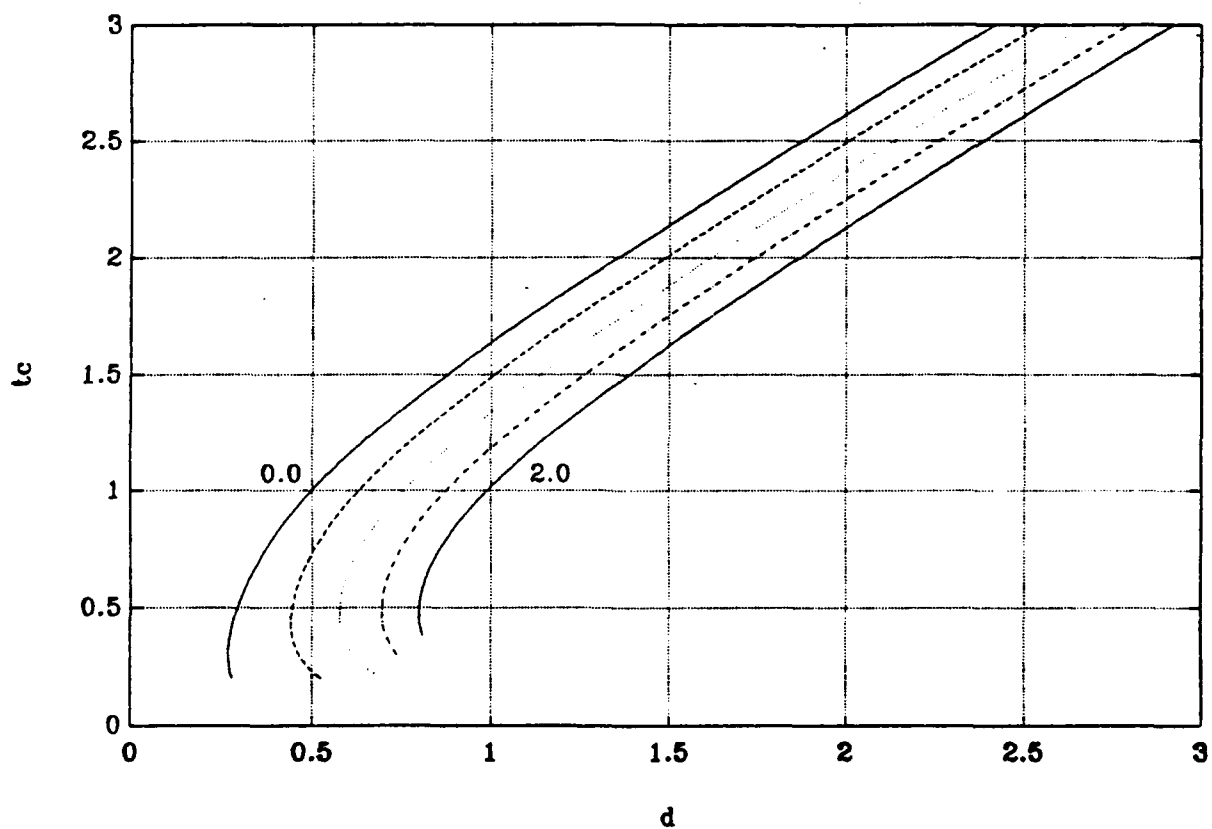


Figure 5. Third Order Approximation For Time Lag:
Critical Visibility Distance d versus t_c for
Time Lags $T = 0.0, 0.5, 1.0, 1.5, 2.0$ sec.

case. The results of the critical visibility d versus the controller time constant t_c for time lags of 0.0, 0.5, 1.0, 1.5 and 2.0 seconds are shown. As with the first and second order cases, the higher values of t_c require higher lookahead distances d for path accuracy, high values of d correspond to a slow guidance law with a loss in speed of response and path accuracy, and the stability curve shifts to the right with increasing values of time lag. However, due to being more accurate than either of the other two cases, the third order approximation provides the tightest stability curves for the same time lags.

F. COMPARISON OF RESULTS

Results of the critical visibility d versus the controller time constant t_c for first, second and third order approximations for time lag have been obtained. Figures 3, 4 and 5 presented the stability curves for first, second and third order approximations for time lags of 0.0, 0.5, 1.0, 1.5 and 2.0 secs. These figures have shown that stability of the control scheme decreases with increasing time lag, and for the same time constant, the critical visibility distance increases with increasing time lag.

Figures 6, 7, 8 and 9 present a comparison of first, second and third order approximations for each time lag considered. These figures show that for each time lag, the third order model presents the largest region of stability for

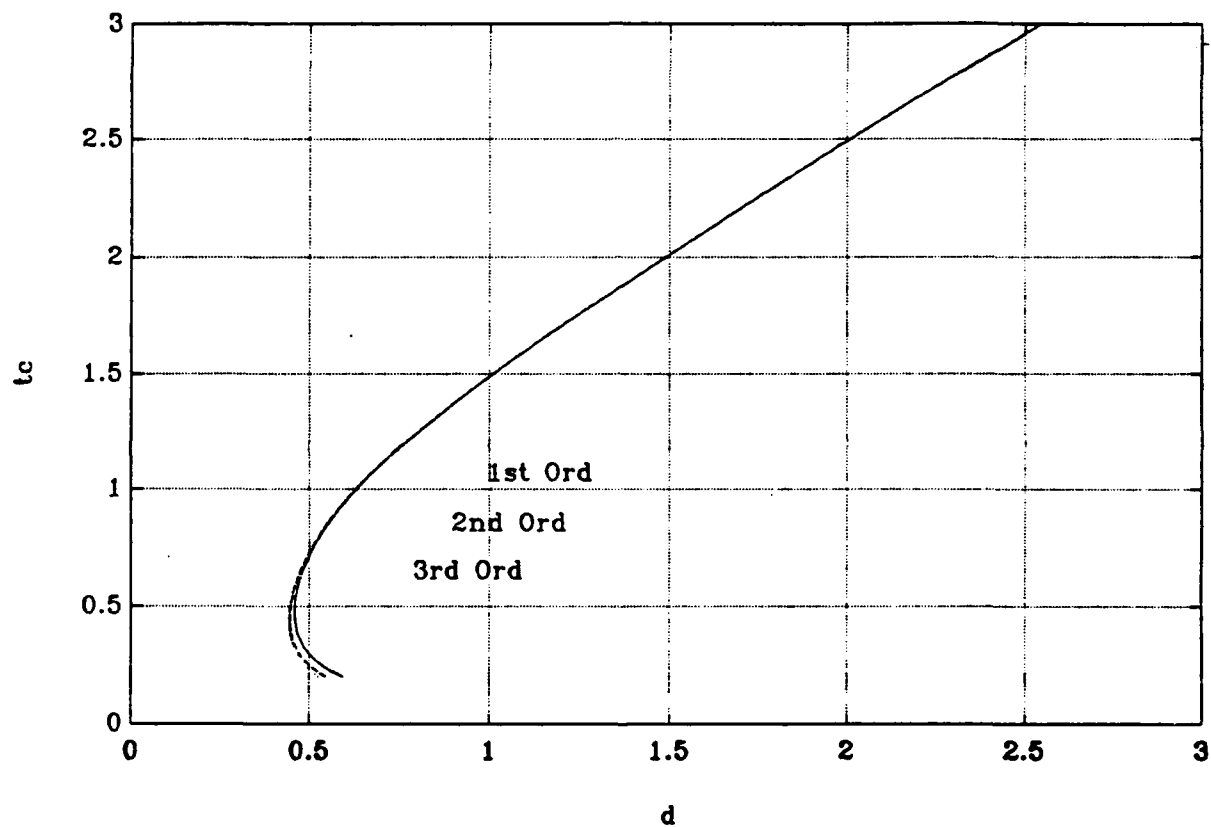


Figure 6. Critical Visibility Distance d versus t_c for First, Second and Third Order Approximation Cases and Time Lag $T = 0.5$ sec.

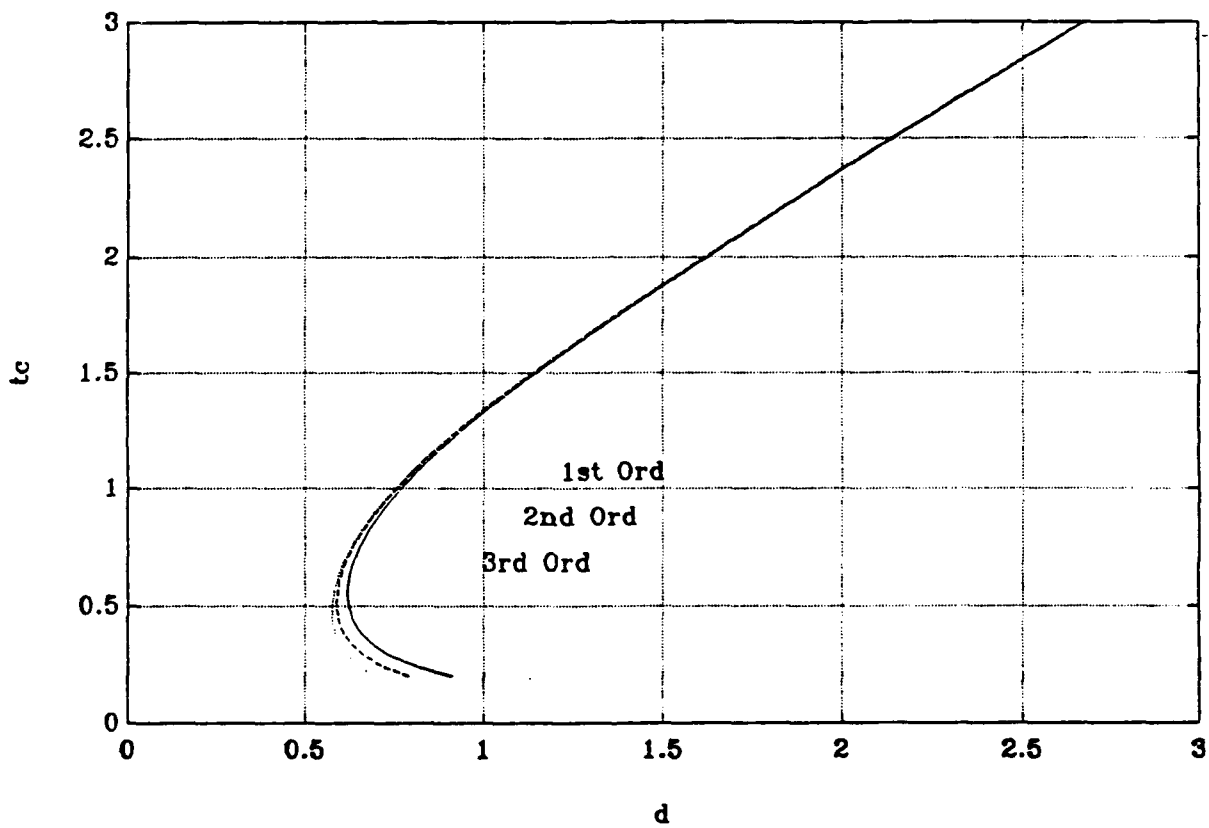


Figure 7. Critical Visibility Distance d versus t_c for First, Second and Third Order Approximation Cases and Time Lag $T = 1.0$ sec.

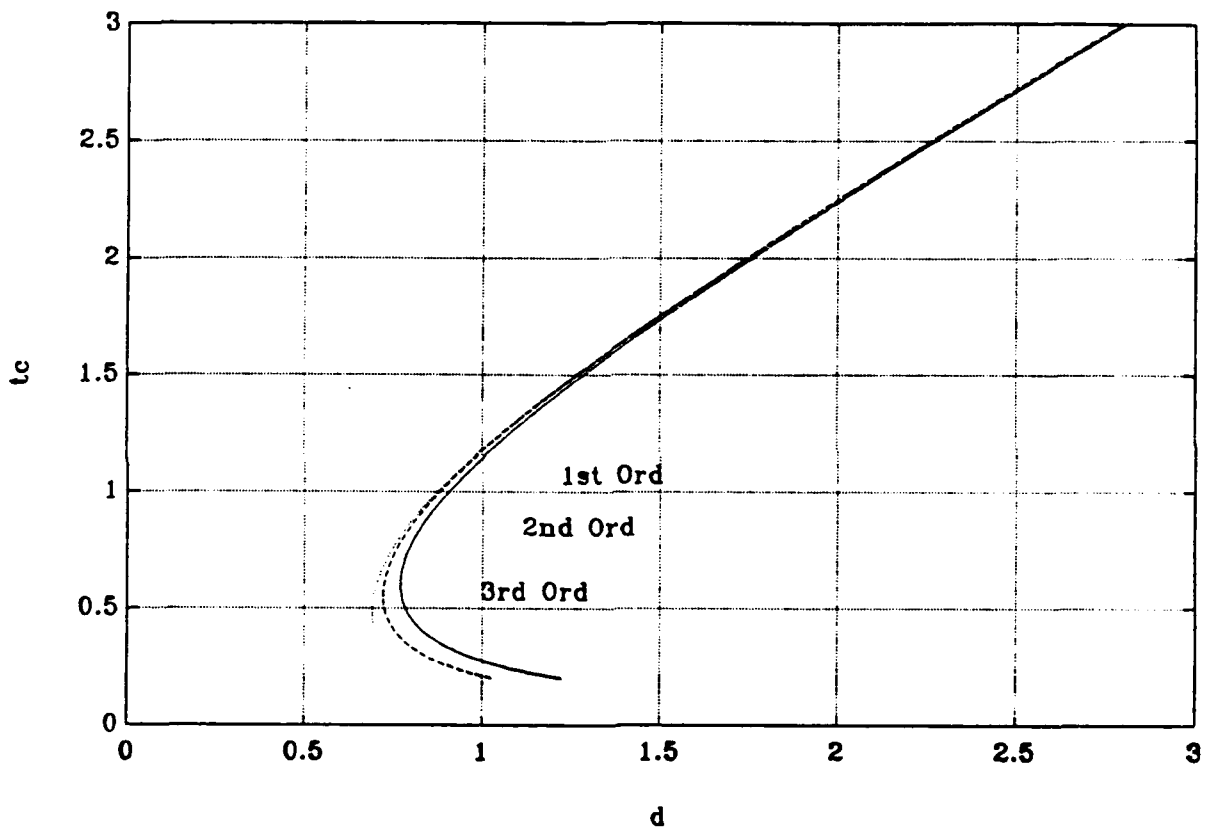


Figure 8. Critical Visibility Distance d versus t_c for First, Second and Third Order Approximation Cases and Time Lag $T = 1.5$ sec.

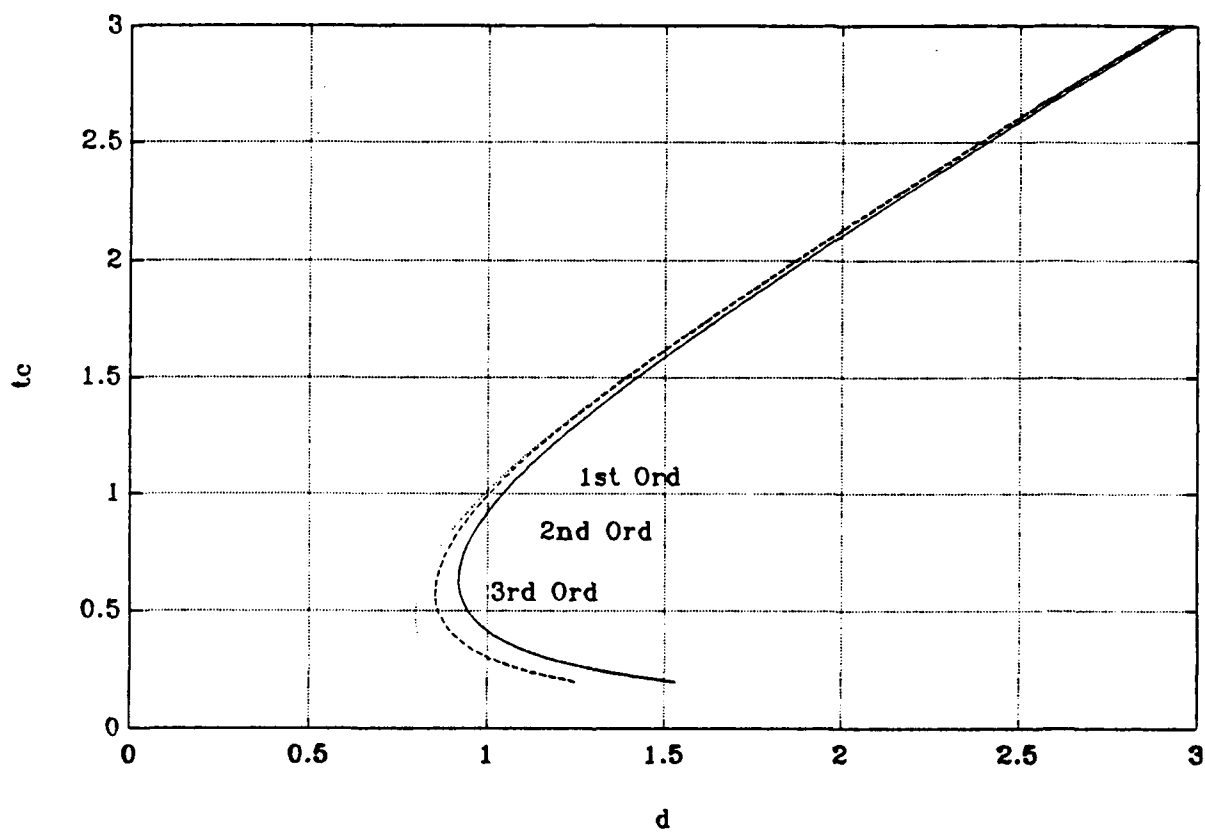


Figure 9. Critical Visibility Distance d versus t_c for First, Second and Third Order Approximation Cases and Time Lag $T = 2.0$ sec.

vehicle straight line path accuracy. In addition, for the same time constant, the critical visibility is least for the third order case. This demonstrates that the minimum lookahead distance required for straight line motion stability decreases with controller time lag accuracy.

It can be seen that the first order approximation is the most conservative for design purposes since it predicts the highest required minimum visibility distance. As expected, the differences among the three approximations are more pronounced as the time lag increases and as the controller time constant becomes smaller; i.e., tighter control.

Figures 10, 11, 12 and 13 present the differences in critical visibilities among first, second and third order approximations for each individual time lag considered. A comparison of these figures demonstrates that the difference in critical visibility between respective orders (time lag models) decreases with decreasing time lag. This indicates that for low time lags, controller time lag accuracy is not as crucial for maintaining vehicle straight line path accuracy as that of higher time lags. The greater the time lag, the greater the difference in critical visibilities among time lag models, and the more crucial is the controller time lag accuracy for stability.

In general, if the guidance law is slow enough to allow sufficient time for the controller to react, stability of straight line motion is guaranteed [Ref. 1].

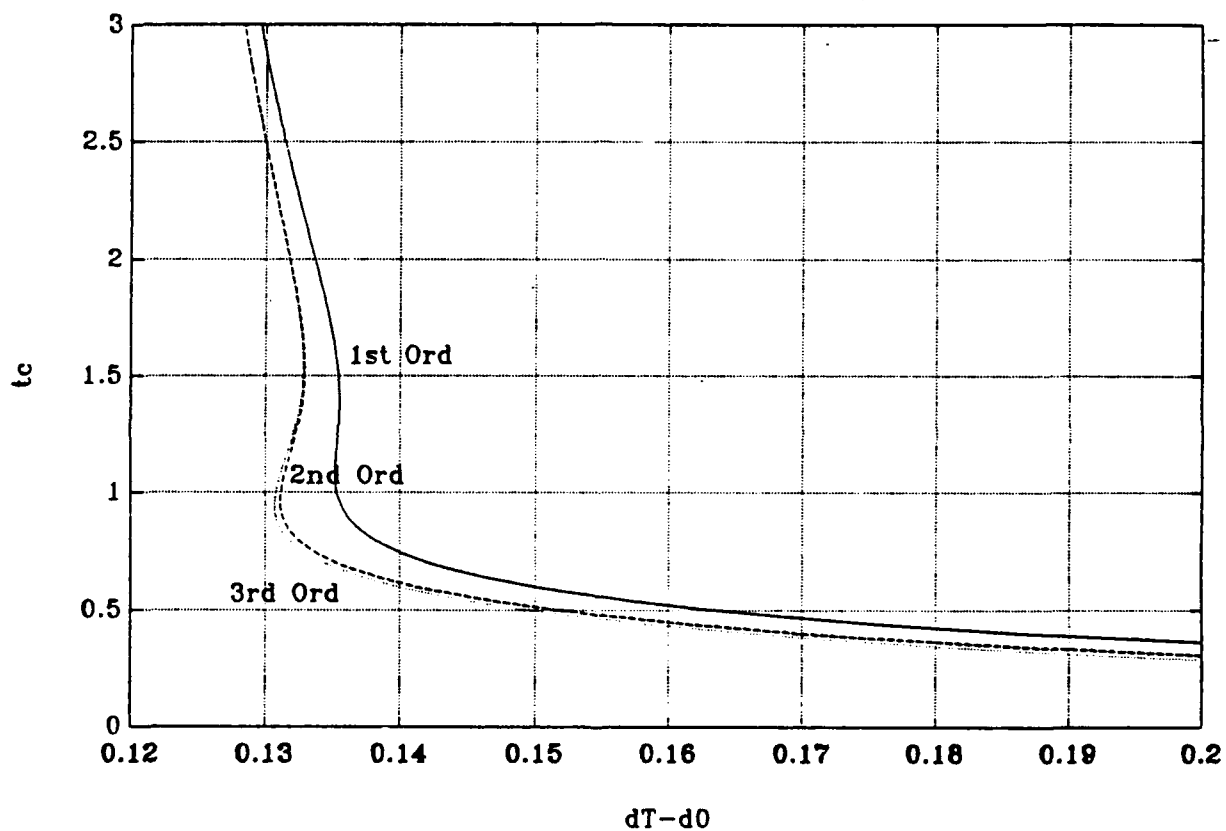


Figure 10. Difference of Critical Visibility Distances at Time Lags of $T = 0.5$ and 0.0 sec versus t_c for First, Second and Third Order Cases.

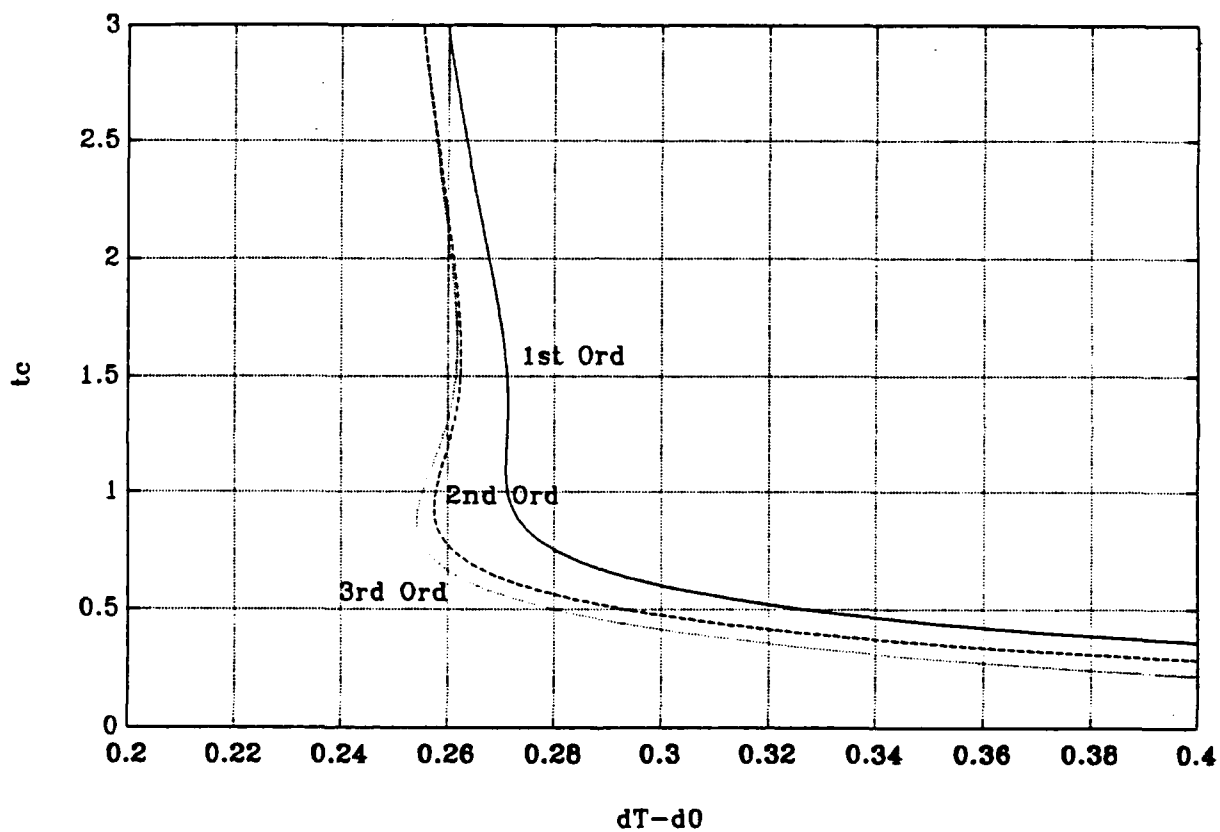


Figure 11. Difference of Critical Visibility Distances at Time Lags of $T = 1.0$ and 0.0 sec versus t_c for First, Second and Third Order Cases.

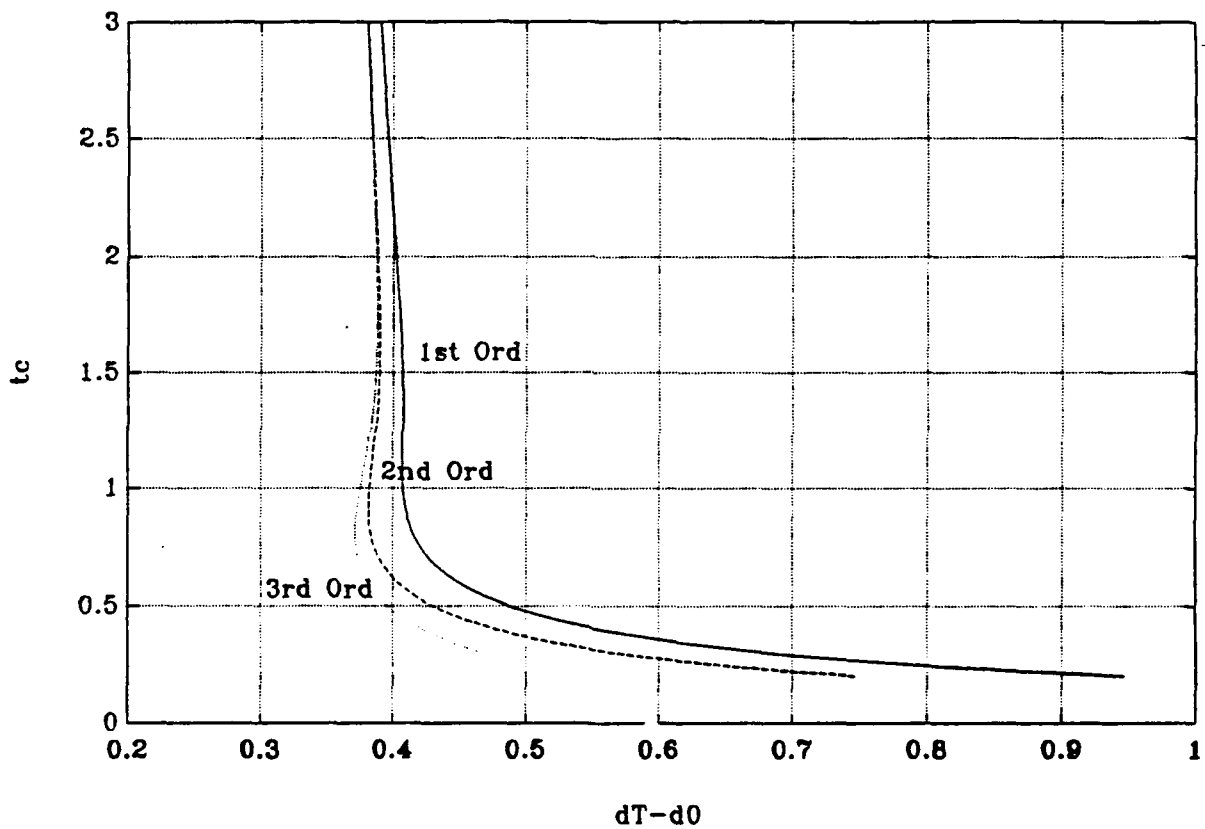


Figure 12. Difference of Critical Visibility Distances at Time Lags of $T = 1.5$ and 0.0 sec versus t_c for First, Second and Third Order Cases.

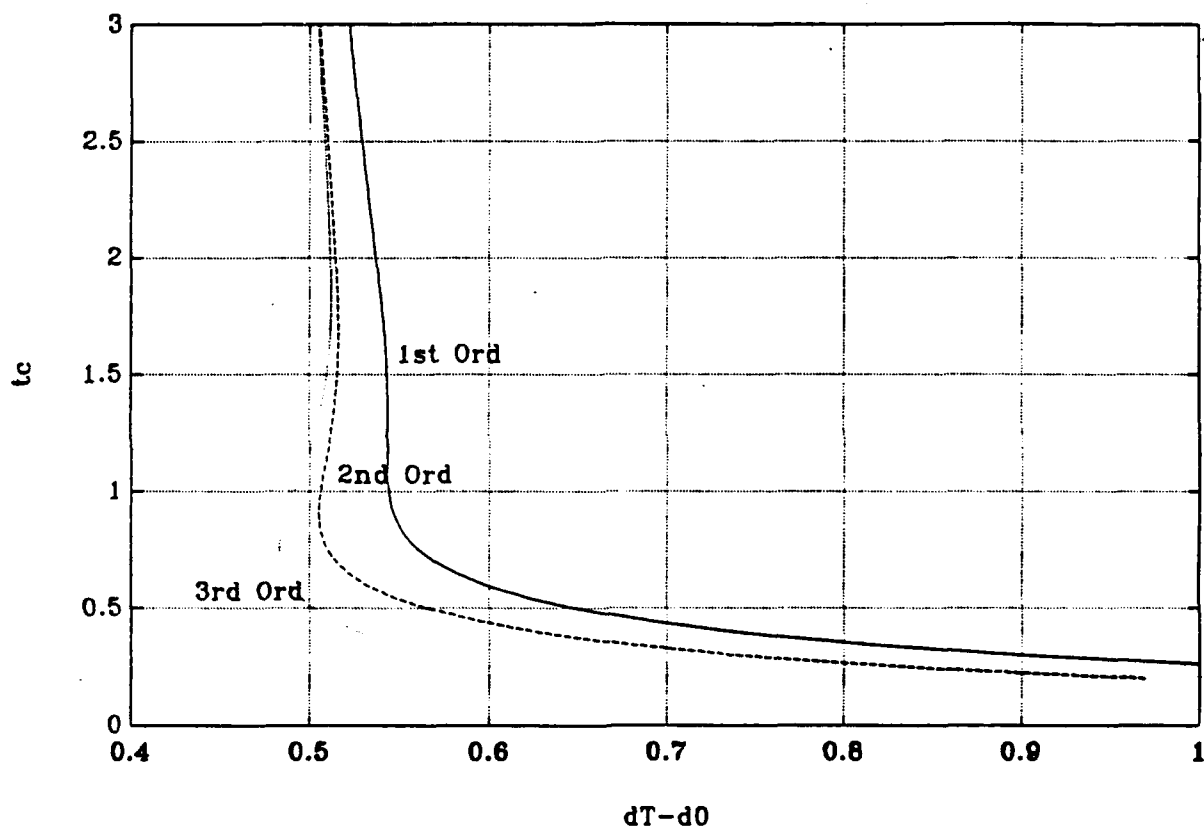


Figure 13. Difference of Critical Visibility Distances at Time Lags of $T = 2.0$ and 0.0 sec versus t_c for First, Second and Third Order Cases.

G. NORMALIZATION OF RESULTS

In an attempt to characterize the stability analysis results for all values of time lag, we proceed as follows: If the vehicle side slip velocity v is very small so that it can be neglected, the equations of motion (2.7a), (2.7b), (2.7c), and (2.7d), take the form

$$\dot{\psi} = r$$

$$\dot{r} = a_{22}ur + b_2u^2\delta$$

$$\dot{y} = u\psi$$

since $v = 0$. The control law is

$$\delta = k_1(\psi - \psi_c) + k_3r,$$

where the first order approximation for the commanded heading angle is

$$\psi_c = -\frac{y - T\dot{y}}{d}.$$

Based on the above equations, we can get the characteristic

equation of the system as

$$s^3 - (a_{22}u + b_2k_3u^2)s^2 - (b_2k_1u^2 - b_2k_1u^3T\frac{1}{d})s - b_2k_1u^3\frac{1}{d} = 0.$$

Applying Routh's criterion to this cubic equation, we find that loss of stability occurs when

$$(a_{22}u + b_2k_3u^2)(b_2k_1u^2 - b_2k_1u^3T\frac{1}{d}) = -b_2k_1u^3\frac{1}{d},$$

from which we can get the critical visibility distance

$$d = -\frac{1}{a_{22} + b_2k_3u} + Tu \quad (3.27)$$

Equation (3.27) can be written as

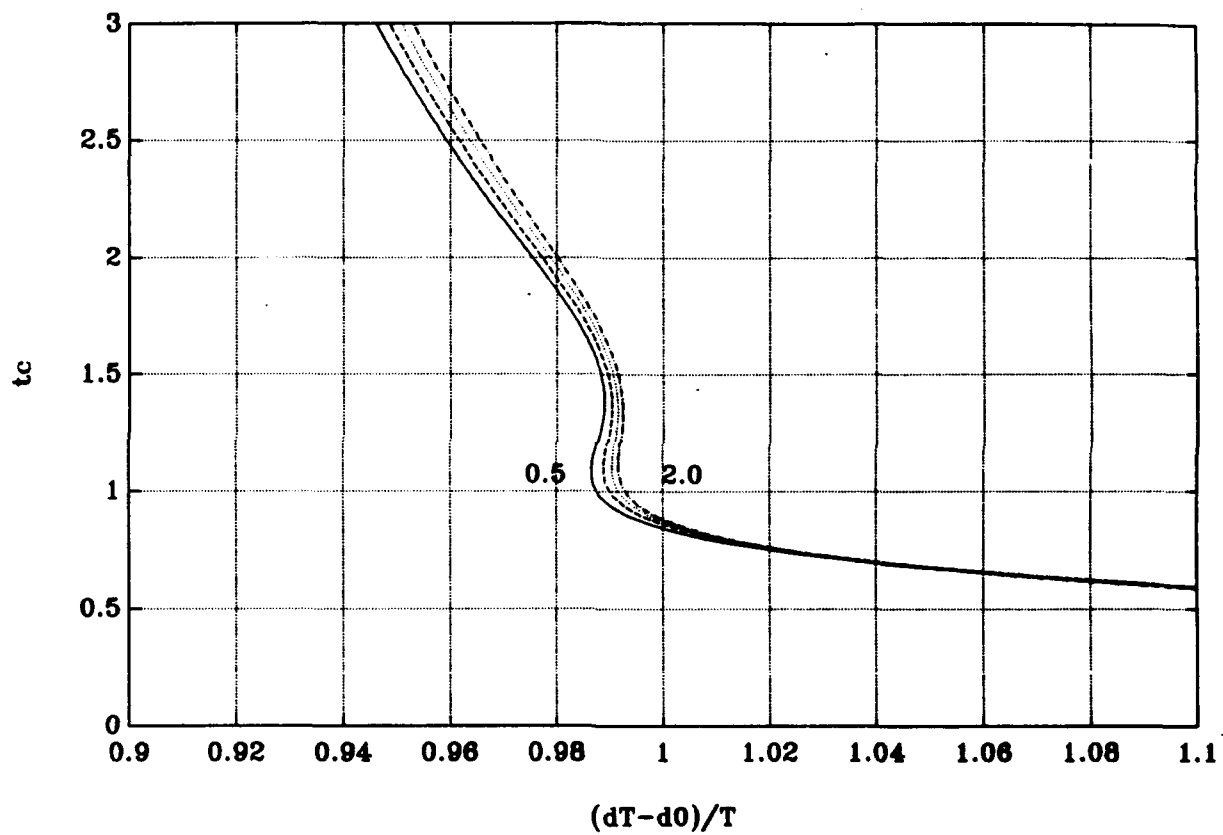
$$\frac{d_r - d_0}{T} = u, \quad (3.28)$$

where d_0 is the critical visibility distance for $T = 0$, and d_r the critical visibility distance for a nonzero T . In dimensionless quantities, equation (3.28) is

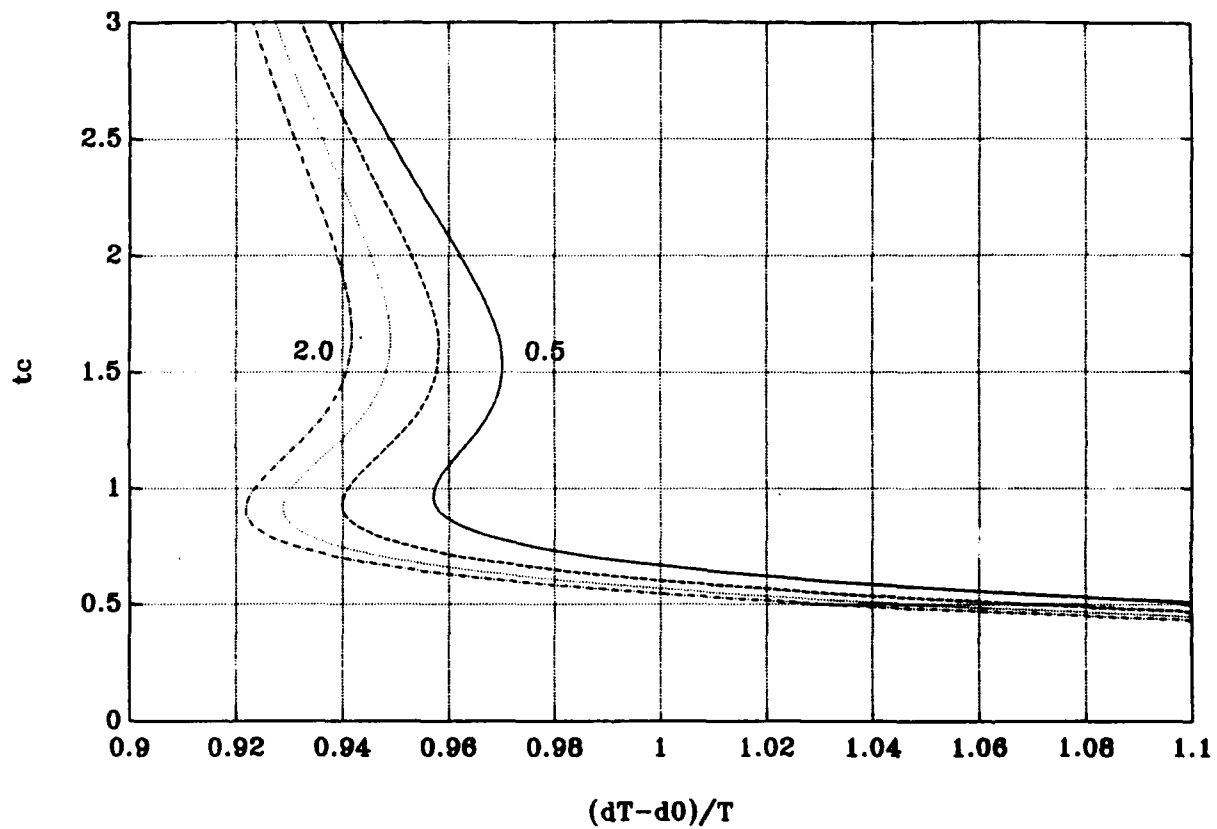
$$\frac{d_T - d_0}{T} = 1 \quad (3.29)$$

where $u = 2.0$ ft/sec is the vehicle speed, and $l = 7.3$ ft is the vehicle length.

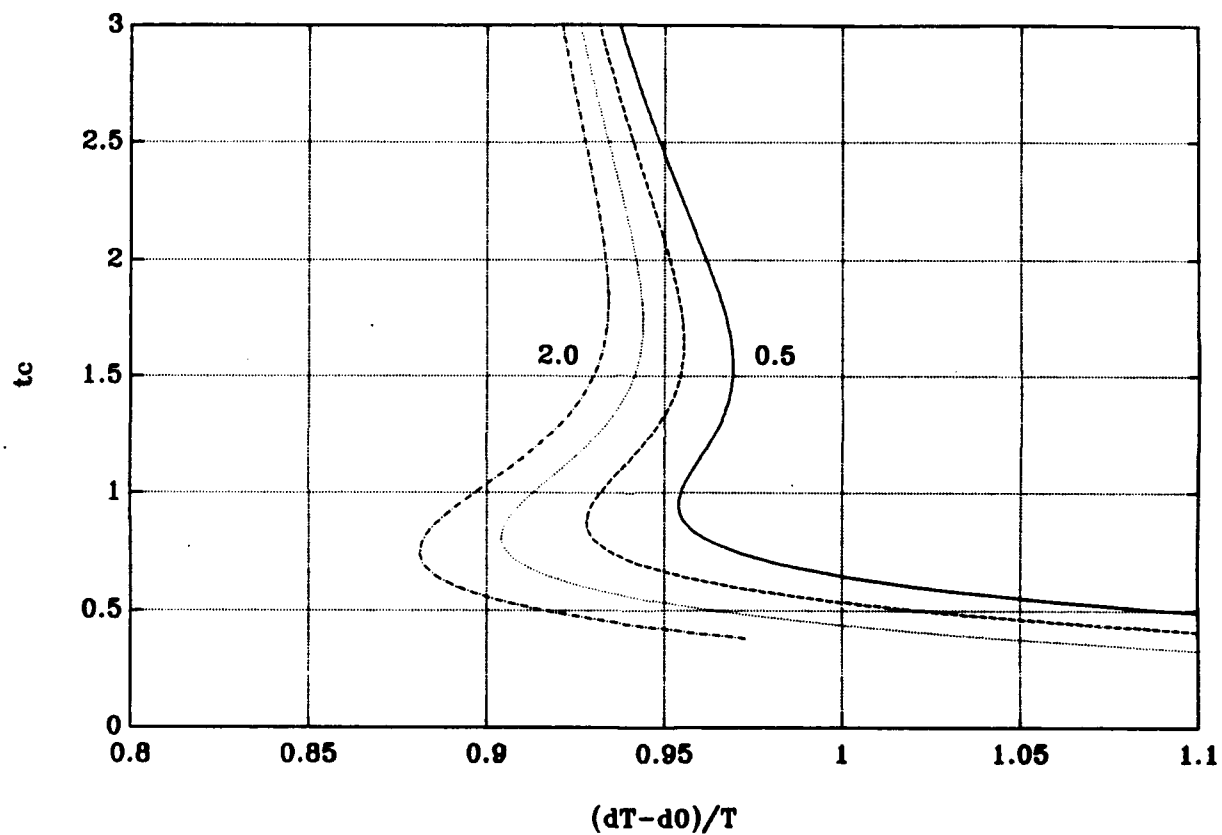
A comparison among the approximate expression (3.29) and the first, second, and third order approximations is shown in figures 14, 15 and 16. The agreement is better for the first order approximation, as expected, and also for the higher controller time constant t_c . This is because a high t_c results in soft vehicle response with negligible amounts of side slip.



**Figure 14. Normalized Critical Visibility versus t_c for
First Order Approximation For Time Lags
 $T = 0.5, 1.0, 1.5, 2.0$ secs.**



**Figure 15. Normalized Critical Visibility versus t_c for
Second Order Approximation For Time Lags
 $T = 0.5, 1.0, 1.5, 2.0$ secs.**



**Figure 16. Normalized Critical Visibility versus t_c for
Third Order Approximation For Time Lags
 $T = 0.5, 1.0, 1.5, 2.0$ secs.**

IV. FREQUENCY RESPONSE ANALYSIS

A. INTRODUCTION

The previous analysis using Taylor expansions of $y(t-T)$ breaks down for approximations beyond third order. The reason for this is that for higher than third order expansions, the matrix B in the generalized eigenvalue problem (3.23) becomes singular. Therefore, it was felt that a different technique should be sought in order to obtain an exact computation of the stability curves and also to check the validity of our calculations. In this chapter, we present a technique based on frequency response methods, which utilizes the Nyquist criterion for stability. The resulting equation is solved using Newton's iteration.

B. NYQUIST CRITERION

From our previously developed model, the linearized horizontal equations of motion are

$$\psi = r \quad (2.7a)$$

$$\dot{v} = a_{11}uv + a_{12}ur + b_1u^2\delta \quad (2.7b)$$

$$\dot{r} = a_{12}uv + a_{22}ur + b_2u^2\delta \quad (2.7c)$$

$$\dot{y} = u\psi + v \quad (2.7d)$$

at any nominal u . The control law is

$$\delta = k_1(\psi - \psi_c) + k_2v + k_3r \quad (2.10)$$

where the commanded heading angle is

$$\psi_c = -\frac{y(t-T)}{d} \quad (3.9)$$

The Laplace transform of equation (3.9) is

$$\psi_c = -\frac{1}{d}e^{-Ts} \quad (4.1)$$

and the resultant rudder control law (2.10) becomes

$$\delta = k_1\psi + k_2v + k_3r + \frac{k_1}{d}ye^{-Ts} \quad (4.2)$$

Following some algebra, the resultant linearized equations of

motion (2.7a), (2.7b), (2.7c), (2.7d) become

$$\dot{\psi} = r \quad (4.3a)$$

$$\dot{v} = (b_1 u^2 k_1 - b_1 u^3 \frac{k_1}{d} T) \psi + (a_{11} u + b_1 u^2 k_2 - b_1 u^2 \frac{k_1}{d} T) v + (a_{12} u + b_1 u^2 k_3) r + b_1 u^2 \frac{k_1}{d} y e^{-\tau s} \quad (4.3b)$$

$$\dot{r} = (b_2 u^2 k_1 - b_2 u^3 \frac{k_1}{d} T) \psi + (a_{21} u + b_2 u^2 k_2 - b_2 u^2 \frac{k_1}{d} T) v + (a_{22} u + b_2 u^2 k_3) r + b_2 u^2 \frac{k_1}{d} y e^{-\tau s} \quad (4.3c)$$

$$\dot{y} = u \psi + v \quad (4.3d)$$

Equations (4.3a), (4.3b), (4.3c), (4.3d) reduce to the state space form

$$\dot{x} = Ax \quad (3.16)$$

The matrix form becomes:

$$\begin{bmatrix} \dot{\psi} \\ \dot{v} \\ \dot{r} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ (b_1 u^2 k_1) & (a_{11} u + b_1 u^2 k_2) & (a_{12} u + b_1 u^2 k_3) & (b_1 u^2 \frac{k_1}{d} e^{-\tau s}) \\ (b_2 u^2 k_1) & (a_{21} u + b_2 u^2 k_2) & (a_{22} u + b_2 u^2 k_3) & (b_2 u^2 \frac{k_1}{d} e^{-\tau s}) \\ u & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \psi \\ v \\ r \\ y \end{bmatrix} \quad (4.4)$$

The characteristic equation of the form $[A-Is] = 0$ becomes after a considerable amount of algebra

$$s^4 + A_3 s^3 + A_2 s^2 + A_1 s + (B_2 s^2 + B_1 s + B_0) De^{-Ts} = 0 \quad (4.5)$$

where

$$A_3 = -(a_{11} + a_{22})u - (b_1 k_2 + b_2 k_3)u^2$$

$$A_2 = (a_{11}a_{22} - a_{12}a_{21})u^2 + (a_{11}b_2 - a_{21}b_1)u^3 k_3 + (a_{22}b_1 - a_{12}b_2)u^3 k_2 - b_2 u^2 k_1$$

$$A_1 = -(a_{12}b_2 - a_{22}b_1)u^3 k_1 De^{-Ts} - (a_{21}b_1 - a_{11}b_2)u^3 k_1$$

$$B_2 = -b_1 u^2 k_1$$

$$B_1 = b_2 u^3 k_1$$

$$B_0 = (a_{11}b_2 - a_{21}b_1)u^4 k_1$$

and

$$D = \frac{1}{d}$$

The characteristic equation of the system is written as

$$1 + K G(s) = 0 \quad (4.6)$$

where

$$G(s) = \frac{(B_2 s^2 + B_1 s + B_0) e^{-Ts}}{s(s^3 + A_3 s^2 + A_2 s + A_1)} \quad (4.7)$$

is the transfer function, and we have denoted

$$K = D \quad (4.8)$$

With $s=j\omega$, the phase angle ϕ is represented by

$$\phi = \angle G(j\omega) \quad (4.9)$$

and it follows that

$$\phi = \angle(e^{-Tj\omega}) - \angle(j\omega) + \angle(-B_2\omega^2 + B_1j\omega + B_0) - \angle(-j\omega^3 - A_3\omega^2 + A_2j\omega + A_1)$$

$$\phi = -\omega T - \frac{\pi}{2} + \tan^{-1}\left(\frac{B_1\omega}{B_0 - B_2\omega^2}\right) - \tan^{-1}\left(\frac{-\omega^3 + A_2\omega}{-A_3\omega^2 + A_1}\right) \quad (4.10)$$

For stability, the Nyquist criterion states (Figure 17) that at the solution of the equation

$$\phi(\omega) = -\pi \quad (4.11)$$

which is phase cross over frequency $\omega = \omega_1$, the gain margin must be equal to 1

$$|KG(j\omega_1)| = 1 \quad (4.12)$$

where $K = D = 1/d$. It follows that equation (4.11) becomes

$$-\omega_1 T - \frac{\pi}{2} + \tan^{-1}\left(\frac{B_1 \omega_1}{B_0 - B_2 \omega_1^2}\right) - \tan^{-1}\left(\frac{-\omega_1^3 + A_2 \omega_1}{-A_3 \omega_1^2 + A_1}\right) = -\pi \quad (4.13)$$

Rearranging, we get

$$\tan^{-1}\left(\frac{B_1 \omega_1}{B_0 - B_2 \omega_1^2}\right) - \tan^{-1}\left(\frac{-\omega_1^3 + A_2 \omega_1}{-A_3 \omega_1^2 + A_1}\right) = -\pi + \frac{\pi}{2} + \omega_1 T$$

Taking the tangent of both sides and using the identity

$$\tan(x-y) = \frac{\tan(x) - \tan(y)}{1 + \tan(x) \tan(y)}$$

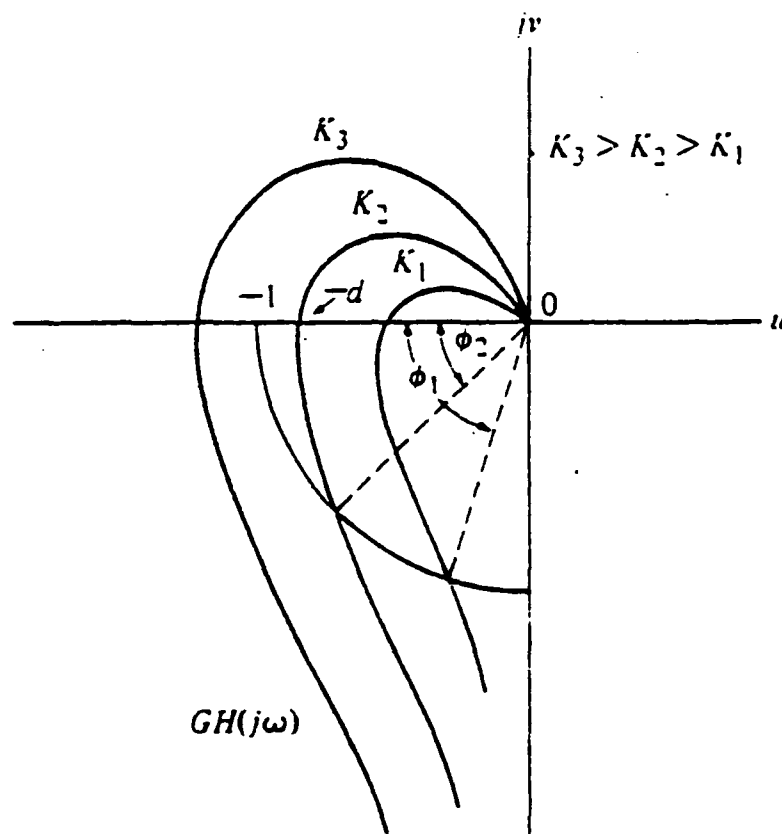


Figure 17. Example of the Nyquist Stability Criterion For Three Different Values of Gain [Ref. 7].

and rearranging following some algebra, the result becomes

$$\frac{B_1 \omega_1 (A_1 - A_3 \omega_1^2) - (B_0 - B_2 \omega_1^2) (-\omega_1^3 + A_2 \omega_1)}{(B_0 - B_2 \omega_1^2) (-A_3 \omega_1^2 + A_1) + B_1 \omega_1 (-\omega_1^3 + A_2 \omega_1)} = -\frac{1}{\tan(\omega_1 T)} \quad (4.14)$$

From equation (4.12), it follows that

$$\frac{1}{d} \frac{|-B_2 \omega_1^2 + B_1 j \omega_1 + B_0|}{\omega_1 |-j \omega_1^3 - A_3 \omega_1^2 + A_2 j \omega_1 + A_1|} = 1$$

Solving for d, we get

$$d = \frac{\sqrt{(B_0 - B_2 \omega_1^2)^2 + B_1^2 \omega_1^2}}{\omega_1 \sqrt{(A_1 - A_3 \omega_1^2)^2 + (A_2 \omega_1 - \omega_1^3)^2}} \quad (4.15)$$

With a time lag $T = 0$, we get from equation (4.14)

$$(B_0 - B_2 \omega_1^2) (A_3 \omega_1^2 + A_1) + B_1 \omega_1 (-\omega_1^3 + A_2 \omega_1) = 0$$

Rearranging, we get

$$(B_2 A_3 - B_1) \omega_1^4 + (B_1 A_2 - B_2 A_1 - B_0 A_3) \omega_1^2 + B_0 A_1 = 0 \quad (4.16)$$

Equation (4.16) can be solved exactly for Q_1 and then d can be computed from (4.15). This zero time lag calculation was performed in order to validate the results of d versus t_c for the zero time lag solution of equation (3.6). The results using the two different techniques were found to be identical.

C. ALGORITHM DESCRIPTION

Introducing the terms

$$\alpha_1 = B_2 A_3 - B_1$$

$$\alpha_2 = B_1 A_2 - B_2 A_1 - B_0 A_3$$

$$\alpha_3 = B_0 A_1$$

and

$$\beta_1 = -B_2$$

$$\beta_3 = B_1 A_1 - B_0 A_2$$

$$\beta_2 = B_0 + B_2 A_2 - B_1 A_3$$

Then equation (4.12) becomes

$$\frac{\beta_1 \omega^5 + \beta_2 \omega^3 + \beta_3 \omega}{\alpha_1 \omega^4 + \alpha_2 \omega^2 + \alpha_3} = -\frac{1}{\tan(\omega T)} \quad (4.17)$$

Rearranging, we get

$$(\beta_1 \omega^5 + \beta_2 \omega^3 + \beta_3 \omega) \tan(\omega T) + \alpha_1 \omega^4 + \alpha_2 \omega^2 + \alpha_3 = 0 \quad (4.18)$$

Equation (4.18) is now in the form

$$f(\omega) = 0 \quad (4.19)$$

Following Newton's iteration for ω , we have

$$\omega_k = \omega_{k-1} - \frac{f(\omega_{k-1})}{f'(\omega_{k-1})} \quad (4.20)$$

From equation (4.20),

$$\begin{aligned} f(\omega) &= (\beta_1 \omega^5 + \beta_2 \omega^3 + \beta_3 \omega) \sin(\omega T) \\ &\quad + (\alpha_1 \omega^4 + \alpha_2 \omega^2 + \alpha_3) \cos(\omega T) \end{aligned} \quad (4.21)$$

and

$$\begin{aligned}
 f'(\omega) = & (5\beta_1\omega^4 + 3\beta_2\omega^2 + \beta_3) \sin(\omega T) \\
 & + (\beta_1\omega^5 + \beta_2\omega^2 + \beta_3\omega) T \cos(\omega T) \\
 & + (4\alpha_1\omega^3 + 2\alpha_2\omega) \cos(\omega T) \\
 & - (\alpha_1\omega^4 + \alpha_2\omega^2 + \alpha_3) T \sin(\omega T)
 \end{aligned} \tag{4.22}$$

Fortran programming and MATLAB were used in order to develop the stability characteristics for the Newton's iteration method. Appendix D presents the program used to develop the stability characteristics for this case.

Figure 18 shows the resulting stability curves for the Newton's iteration method. The results of the critical visibility d versus the controller time constant t_c for time lags of 0.0, 0.5, 1.0, 1.5, and 2.0 secs are shown.

As with the first, second and third order approximation cases, the higher values of t_c require higher lookahead distances d for path accuracy, high values of d correspond to a slow guidance law with loss in speed of response and path accuracy, and the stability curves shift to the right with increasing values of time lag. However, since the Newton's iteration method presents the exact solution of the frequency response to equation (4.19), the resulting stability curves

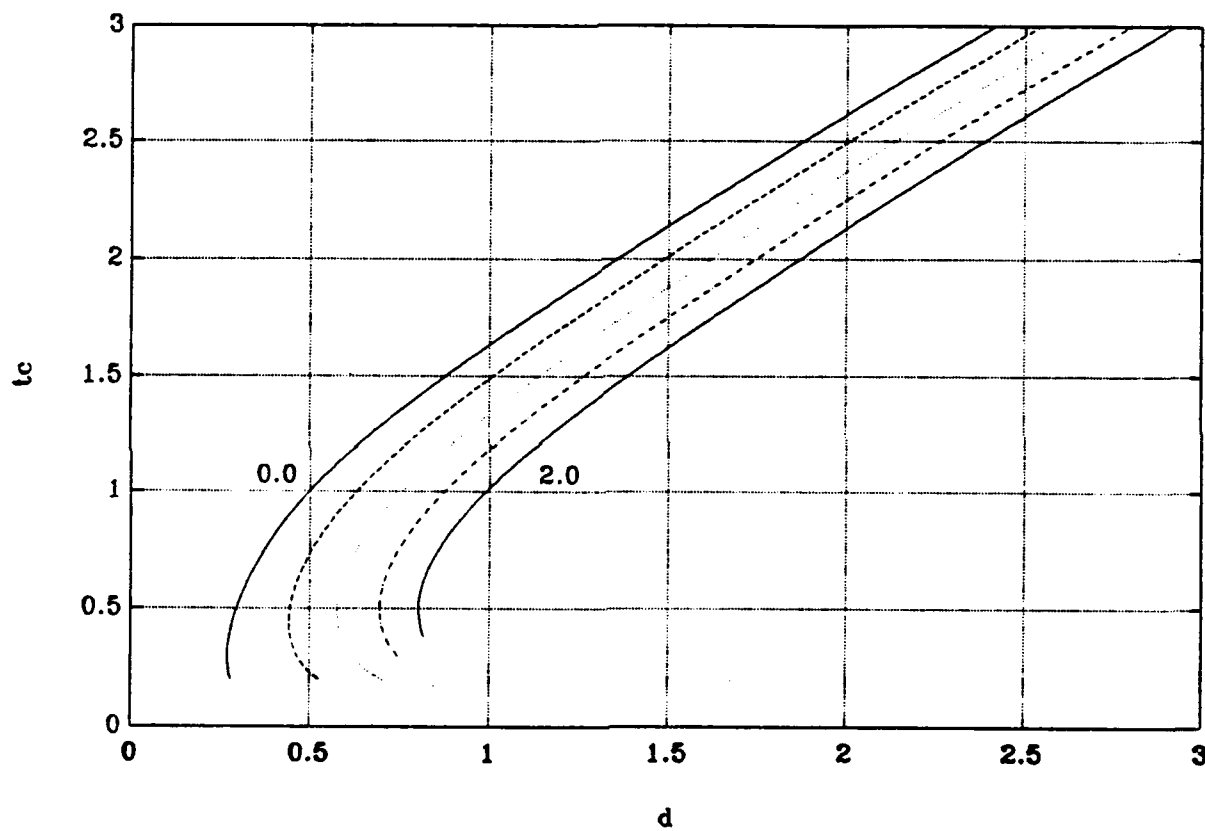


Figure 18. Newton's Iteration Method: Critical Visibility
Distance d versus t_c for Time Lags $T = 0.0,$
 $0.5, 1.0, 2.0$ secs.

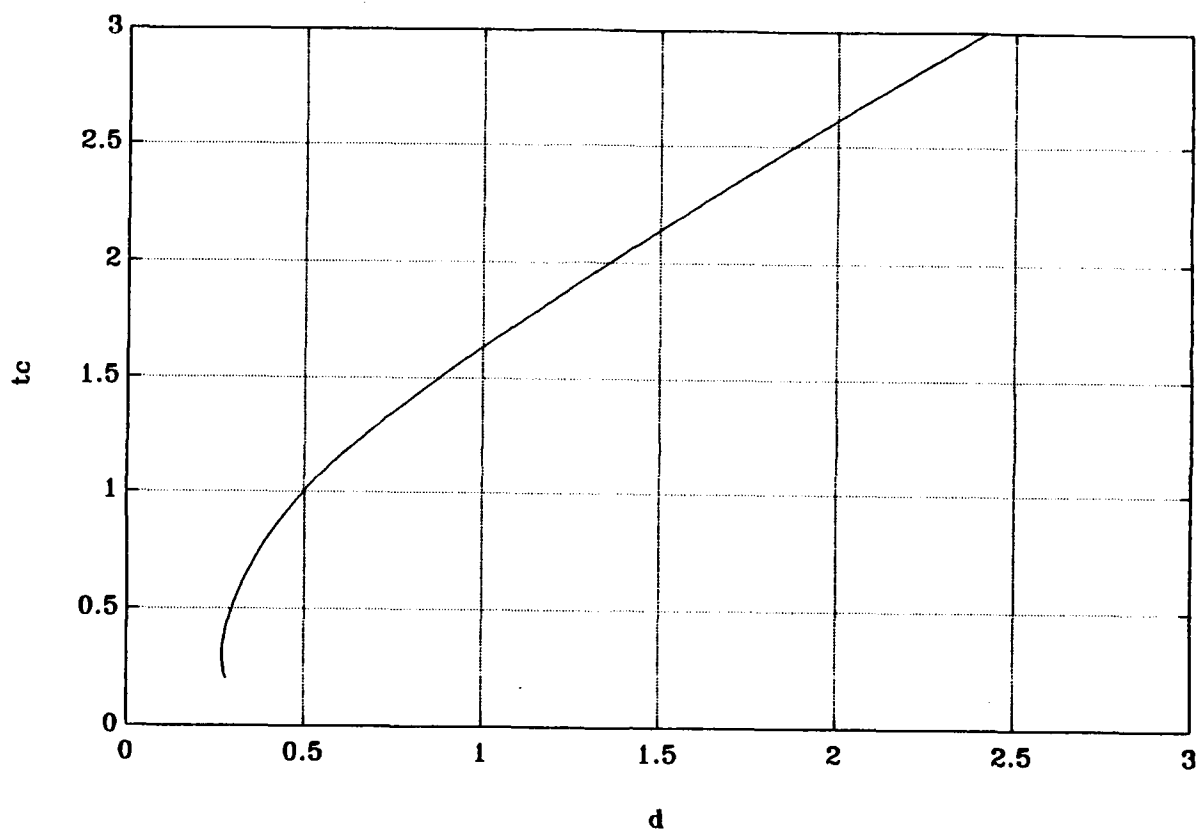
represent the precise stability characteristics for these time lags.

D. RESULTS AND DISCUSSION

Results of the critical visibility d versus the controller time constant t_c for the Newton's iteration method has been demonstrated. Figure 18 presented the precise stability curves for time lags of 0.0, 0.5, 1.0, 1.5 and 2.0 secs.

This figure, as well as those for the first, second and third order approximation cases, demonstrates that the stability of the control scheme decreases with increase in time lag and for the same time constant, critical visibility increases with increase in time lag.

Figures 19, 20, 21, 22 and 23 present a comparison of generated stability curves among the Newton's iteration method and those for the first, second and third order approximation cases for time lags of 0.0, 0.5, 1.0, 1.5 and 2.0 secs. It can be seen that there is barely a slight difference in the stability curves for the Newton's iteration method case and the third order approximation case. This indicates that a third order approximation for time lag presents the best model for accounting for a positional information time lag in the vehicle control law.



**Figure 19. Critical Visibility Distance d versus t_c for
Newton's Iteration Method and First, Second and
Third Order Approximations; $T = 0.0$ sec.**

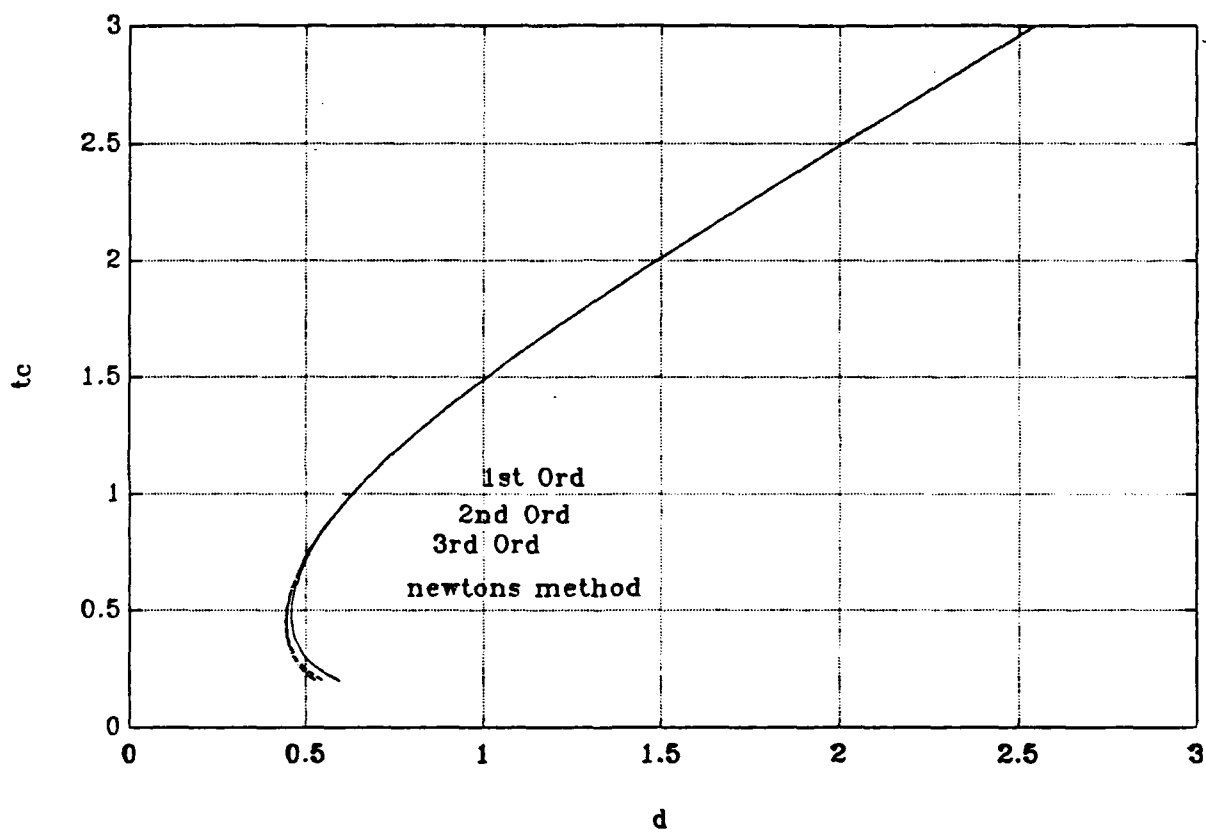


Figure 20. Critical Visibility Distance d versus t_c for Newton's Iteration Method and First, Second and Third Order Approximations; $T = 0.5$ sec.

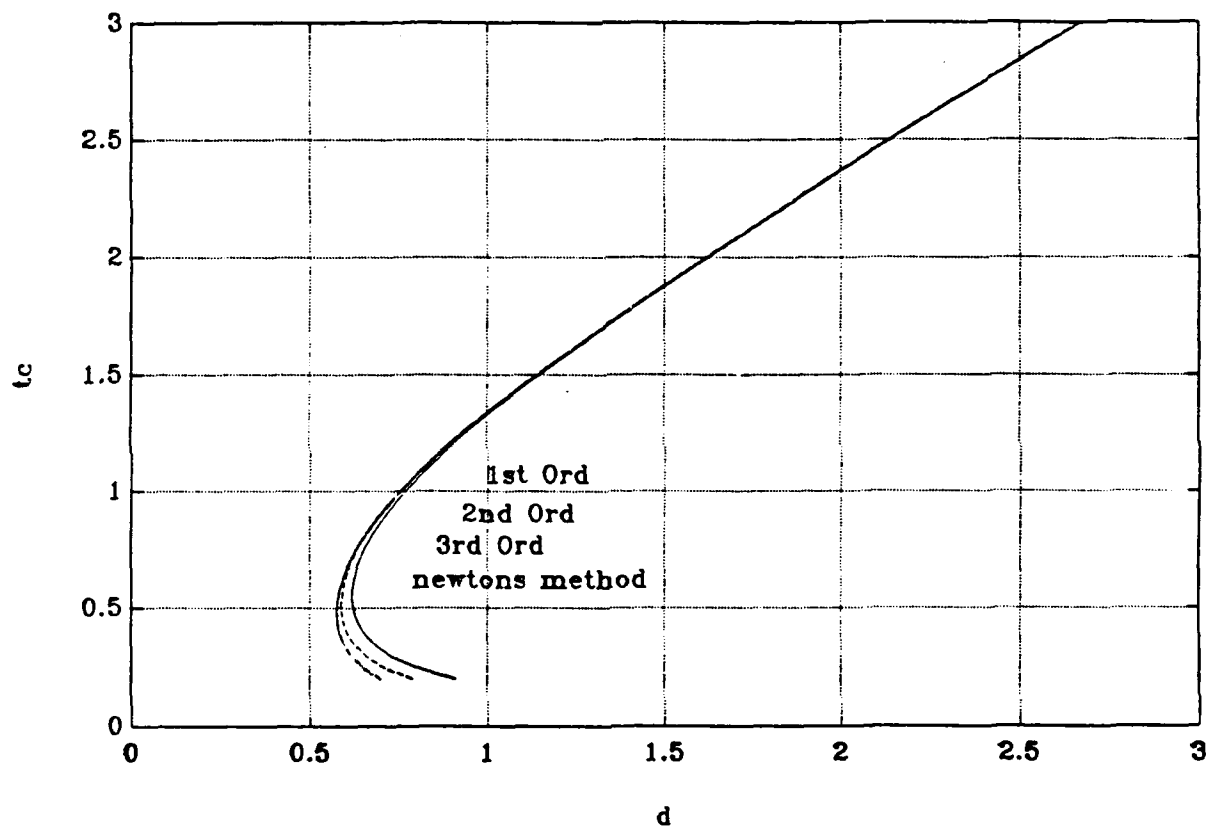


Figure 21. Critical Visibility Distance d versus t_c for Newton's Iteration Method and First, Second and Third Order Approximations; $T = 1.0$ sec.

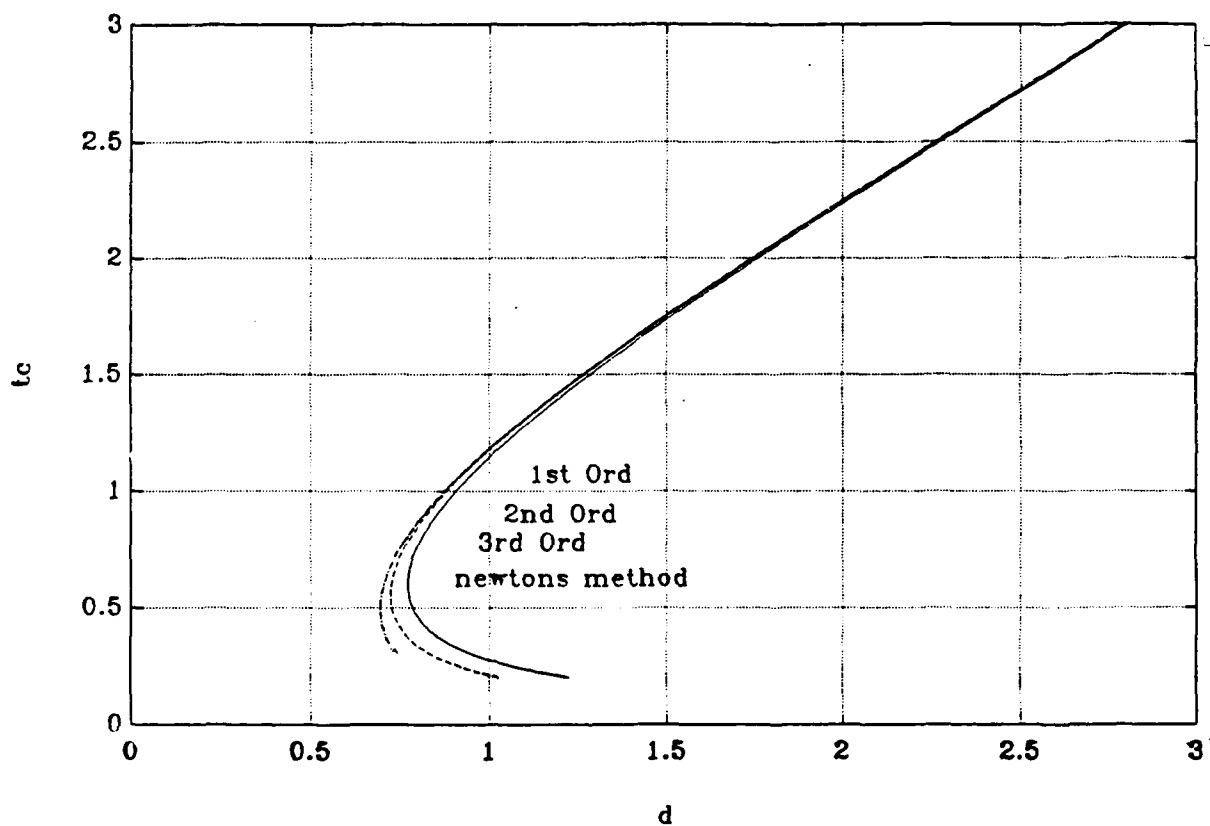


Figure 22. Critical Visibility Distance d versus t_c for Newton's Iteration Method and First, Second and Third Order Approximations; $T = 1.5$ sec.

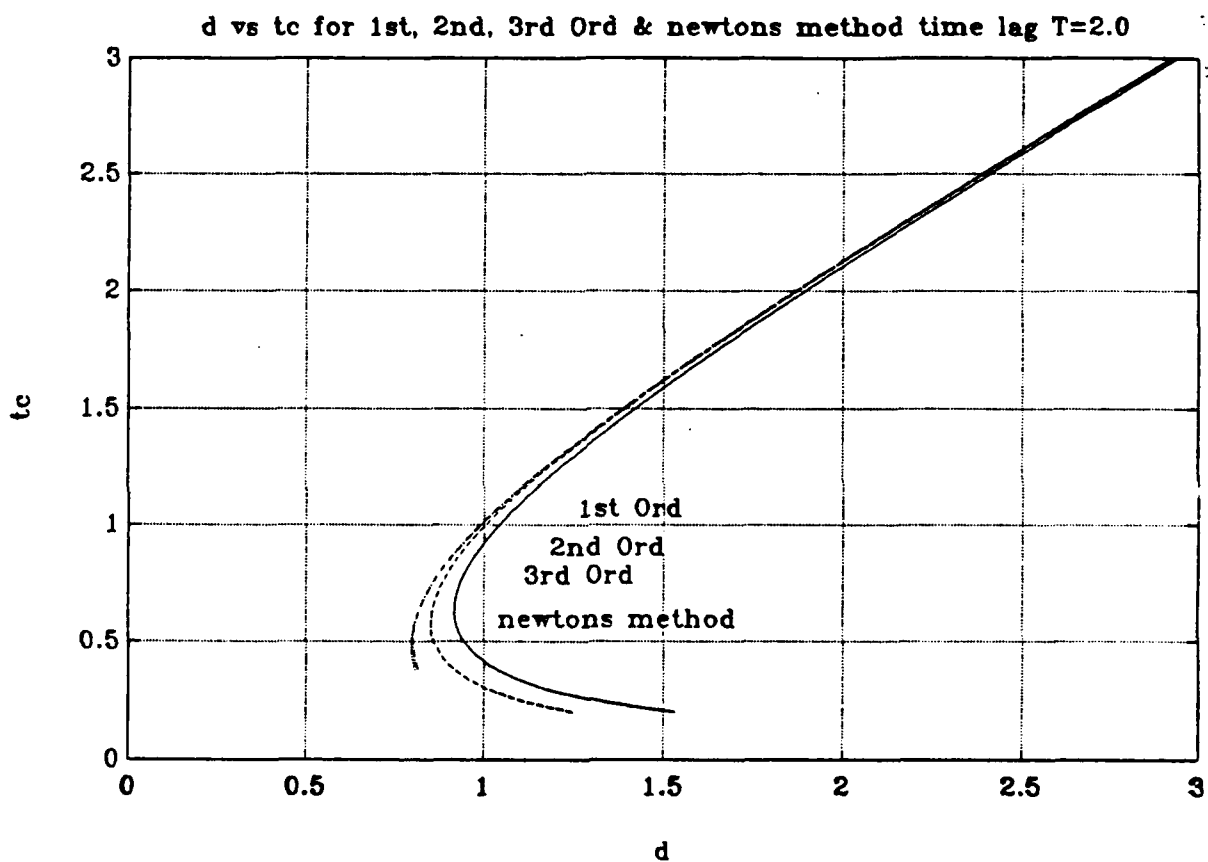


Figure 23. Critical Visibility Distance d versus t_c for Newton's Iteration Method and First, Second and Third Order Approximations; $T = 2.0$ sec.

CONCLUSIONS AND RECOMMENDATIONS

A. CONCLUSIONS

A methodology for considering positional information time lags in the control law for vehicle maneuvering in the horizontal plane has been presented. The relationship of the critical visibility versus the controller time constant for both zero and non-zero time lags has been established.

Time lags have been approximated by first, second and third order Taylor expansions in the commanded vehicle heading in the control law. A comparison of the resulting stability curves has demonstrated that the third order approximation presents the greatest stability and the least critical visibility for a given time lag. It was found that the first order approximation was the most conservative for controller design purposes since it predicted the highest required critical visibility distance. In addition, the differences among the three approximations became more pronounced with increasing time lag and decreasing controller time constant (tighter control).

Further analysis was conducted to characterize the stability results for any value of time lag using a normalization procedure and the first order approximation for time lag in the commanded vehicle heading in the control law.

A comparison of the normalized stability curves for each time lag showed that the greatest differences occur with increasing order and decreasing controller time constant. This is due to a tight (quick) vehicle response with significant side slip at lower time constants.

Frequency response techniques based on the Nyquist stability criterion were used to produce the exact stability curves for comparison with the previously generated Taylor expansion stability curves. These results virtually matched those obtained for the third order approximation time lag case and validated our calculations and stability curves generated for the case of zero time lag. These results indicated that the third order approximation best represents a positional information time lag in the control law, however for design purposes, the first order approximation provides the most conservative approach.

B. RECOMMENDATIONS

Some recommendations for further research are as follows:

- * Experimental verification using the NPS AUV II.
- * Consideration of time lag in the simulation of an Inertial Navigation System required for positional updates.
- * Analysis of positional information time lags in the motion stability in the vertical plane, along curved paths, or with respect to other guidance schemes.

APPENDIX A

```

C      PROGRAM THESIS1.FOR (Time Delay-1st Order Approx)
C      BIFURCATION ANALYSIS
C
C      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
C      DOUBLE PRECISION K1,K2,K3,L,NR,NV,NDRS,NDRB,IZ,MASS,
C      &      NRDOT,NVDOT
C      DIMENSION A(4,4),FV1(4),IV1(4),ZZZ(4,4),WR(4),WI(4)
C
C      OPEN (11,FILE='BIF1.RES',STATUS='NEW')
C      OPEN (12,FILE='BIF2.RES',STATUS='NEW')
C      OPEN (13,FILE='BIF3.RES',STATUS='NEW')
C
C      Vehicle Parameters:
C      WEIGHT=435.0
C      IZ      =45.0
C      L       =7.3
C      RHO     =1.94
C      G       =32.2
C      XG      =0.0104
C      MASS    =WEIGHT/G
C
C      YRDOT =-0.00178*0.5*RHO*L**4
C      YVDOT =-0.03430*0.5*RHO*L**3
C      YR     =+0.01187*0.5*RHO*L**3
C      YV     =-0.03896*0.5*RHO*L**2
C      YDRS   =+0.02345*0.5*RHO*L**2
C      YDRB   =+0.02345*0.5*RHO*L**2
C      NRDOT  =-0.00047*0.5*RHO*L**5
C      NVDOT  =-0.00178*0.5*RHO*L**4
C      NR     =-0.01022*0.5*RHO*L**4
C      NV     =-0.00769*0.5*RHO*L**3
C      NDRS   =-0.337*0.02345*0.5*RHO*L**3
C      NDRB   =+0.283*0.02345*0.5*RHO*L**3
C
C      WRITE (*,1001)
C      READ  (*,*)      TCMIN,TCMAX,ITC
C      WRITE (*,1002)
C      READ  (*,*)      XDMIN,XDMAX,IXD
C      XDMIN=XDMIN*L
C      XDMAX=XDMAX*L
C      WRITE (*,1003)
C      READ  (*,*)      U
C
C      WRITE(*,1100)
C      READ (*,*) TL
C
C      DH = (IZ-NRDOT)*(MASS-YVDOT)-
C      &      (MASS*XG-YRDOT)*(MASS*XG-NVDOT)
C      AA11=((IZ-NRDOT)*YV-(MASS*XG-YRDOT)*NV)/DH
C      AA12=((IZ-NRDOT)*(-MASS+YR)-
C      &      (MASS*XG-YRDOT)*(-MASS*XG+NR))/DH
C      AA21=((MASS-YVDOT)*NV-(MASS*XG-NVDOT)*YV)/DH
C      AA22=((MASS-YVDOT)*(-MASS*XG+NR)-
C      &      (MASS*XG-NVDOT)*(-MASS+YR))/DH
C      BB11=((IZ-NRDOT)*YDRS-(MASS*XG-YRDOT)*NDRS)/DH
C      BB12=((IZ-NRDOT)*YDRB-(MASS*XG-YRDOT)*NDRB)/DH
C      BB21=((MASS-YVDOT)*NDRS-(MASS*XG-NVDOT)*YDRS)/DH
C      BB22=((MASS-YVDOT)*NDRB-(MASS*XG-NVDOT)*YDRB)/DH
C
C      WRITE (*,1004)

```

```

C      READ (*,*)      RATIO
C      BB1=BB11+RATIO*BB12
C      BB2=BB21+RATIO*BB22
C      EPS =1.D-5
C      ILMAX=1500
C      DO 1 I=1,ITC
C        WRITE (*,2001) I,ITC
C        TC=TCMIN+(I-1)*(TCMAX-TCMIN)/(ITC-1)
C        TCD=TC*L/U
C        POLEC=1.0/TCD
C        AD1=3.0*POLEC
C        AD2=3.0*POLEC**2
C        AD3=POLEC**3
C        A1=BB1*U*U
C        B1=BB2*U*U
C        C1=-AD1-(AA11+AA22)*U
C        A2=(BB1*AA22-BB2*AA12)*U**3
C        B2=(BB2*AA11-BB1*AA21)*U**3
C        K1=AD3/((BB2*AA11-BB1*AA21)*U**3)
C        C2=AD2-(AA11*AA22-AA12*AA21)*U**2+BB2*U*U*K1
C        K2=(C1*B2-C2*B1)/(A1*B2-A2*B1)
C        K3=(C2*A1-C1*A2)/(A1*B2-A2*B1)
C        DO 2 J=1,IXD
C          XD=XDMIN+(J-1)*(XDMAX-XDMIN)/(IXD-1)
C
C          CALL LINEAR(K1,K2,K3,U,XD,AA11,AA12,AA21,AA22,BB1,BB2,A,TL)
C          CALL RG(4,4,A,WR,WI,0,ZZZ,IV1,FV1,IERR)
C          CALL DSTABL(DEOS,WR,WI,FREQ)
C
C          IF (J.GT.1) GO TO 10
C          DEOSOO=DEOS
C          XDOO =XD
C          LL=0
C          GO TO 2
10      DEOSNN=DEOS
C          XDNN =XD
C          PR=DEOSNN*DEOSOO
C          IF (PR.GT.0.D0) GO TO 3
C          LL=LL+1
C          IF (LL.GT.3) STOP 1000
C          IL=0
C          XDO=XDOO
C          XDN=XDNN
C          DEOSO=DEOSOO
C          DEOSN=DEOSNN
C          XDL=XDO
C          XDR=XDN
C          DEOSL=DEOSO
C          DEOSR=DEOSN
C          XD=(XDL+XDR)/2.D0
C
C          CALL LINEAR(K1,K2,K3,U,XD,AA11,AA12,AA21,AA22,BB1,BB2,A,TL)
C          CALL RG(4,4,A,WR,WI,0,ZZZ,IV1,FV1,IERR)
C          CALL DSTABL(DEOS,WR,WI,FREQ)
C
C          DEOSM=DEOS
C          XDM=XD

```

```

PRL=DEOSL*DEOSM
PRR=DEOSR*DEOSM
IF (PRL.GT.0.D0) GO TO 5
XDO=XDL
XDN=XDM
DEOSO=DEOSL
DEOSN=DEOSM
IL=IL+1
IF (IL.GT.ILMAX) STOP 3100
DIF=DABS(XDL-XDM)
IF (DIF.GT.EPS) GO TO 6
XD=XDM
GO TO 4
5 IF (PRR.GT.0.D0) STOP 3200
XDO=XDM
XDN=XDR
DEOSO=DEOSM
DEOSN=DEOSR
IL=IL+1
IF (IL.GT.ILMAX) STOP 3100
DIF=DABS(XDM-XDR)
IF (DIF.GT.EPS) GO TO 6
XD=XDM
4 LLL=10+LL
WRITE (LLL,*) XD/L,TC
3 XDOO=XDNN
DEOSOO=DEOSNN
2 CONTINUE
1 CONTINUE
C
1001 FORMAT (' ENTER MIN, MAX, AND INCREMENTS OF TC')
1002 FORMAT (' ENTER MIN, MAX, AND INCREMENTS OF XD')
1003 FORMAT (' ENTER U')
1004 FORMAT (' ENTER BOW/STERN RUDDER RATIO')
1100 FORMAT (' ENTER TIME LAG TL')
2001 FORMAT (2I5)
END
C
SUBROUTINE DSTABL(DEOS,WR,WI,OMEGA)
IMPLICIT DOUBLE PRECISION (A-H,O-Z)
DIMENSION WR(4),WI(4)
DEOS=-1.0D+20
DO 1 I=1,4
IF (WR(I).LT.DEOS) GO TO 1
DEOS=WR(I)
IJ=I
1 CONTINUE
OMEGA=WI(IJ)
OMEGA=DABS(OMEGA)
RETURN
END
C
SUBROUTINE LINEAR(K1,K2,K3,U,XD,AA11,AA12,AA21,AA22,BB1,BB2,A,TL)
IMPLICIT DOUBLE PRECISION (A-H,O-Z)
DOUBLE PRECISION K1,K2,K3
DIMENSION A(4,4)
A(1,1)=0.0D0
A(1,2)=0.0D0
A(1,3)=1.0D0
A(1,4)=0.0D0

```

```

A(2,1)=          BB1*U*U*K1*(XD-U*TL)/XD
A(2,2)=U*(BB1*U*(K2*XD-K1*TL)+AA11*XD)/XD
A(2,3)=AA12*U+BB1*U*U*K3
A(2,4)=          BB1*U*U*K1/XD
A(3,1)=          BB2*U*U*K1*(XD-U*TL)/XD
A(3,2)=U*(BB2*U*(K2*XD-K1*TL)+AA21*XD)/XD
A(3,3)=AA22*U+BB2*U*U*K3
A(3,4)=          BB2*U*U*K1/XD
A(4,1)=U
A(4,2)=1.0D0
A(4,3)=0.0D0
A(4,4)=0.0D0
RETURN
END

```

APPENDIX B

```

C   PROGRAM THESIS2.FOR (Time Delay-2nd Order Approx)
C   BIFURCATION ANALYSIS
C
C   IMPLICIT DOUBLE PRECISION (A-H,O-Z)
C   DOUBLE PRECISION K1,K2,K3,L,NR,NV,NDRS,NDRB,IZ,MASS,
C   & NRDOT,NVDOT
C   DIMENSION A(4,4),FV1(4),IV1(4),ZZZ(4,4),WR(4),WI(4)
C
C   OPEN (11,FILE='BIF1.RES',STATUS='NEW')
C   OPEN (12,FILE='BIF2.RES',STATUS='NEW')
C   OPEN (13,FILE='BIF3.RES',STATUS='NEW')
C
C   Vehicle Parameters:
C   WEIGHT=435.0
C   IZ      =45.0
C   L       =7.3
C   RHO     =1.94
C   G       =32.2
C   XG      =0.0104
C   MASS    =WEIGHT/G
C
C   YRDOT  =-0.00178*0.5*RHO*L**4
C   YVDOT  =-0.03430*0.5*RHO*L**3
C   YR      =+0.01187*0.5*RHO*L**3
C   YV      =-0.03896*0.5*RHO*L**2
C   YDRS    =+0.02345*0.5*RHO*L**2
C   YDRB    =+0.02345*0.5*RHO*L**2
C   NRDOT   =-0.00047*0.5*RHO*L**5
C   NVDOT   =-0.00178*0.5*RHO*L**4
C   NR      =-0.01022*0.5*RHO*L**4
C   NV      =-0.00769*0.5*RHO*L**3
C   NDRS    =-0.337*0.02345*0.5*RHO*L**3
C   NDRB    =+0.283*0.02345*0.5*RHO*L**3
C
C   WRITE (*,1001)
C   READ  (*,*)      TCMIN,TCMAX,ITC
C   WRITE (*,1002)
C   READ  (*,*)      XDMIN,XDMAX,IXD
C   XDMIN=XDMIN*L
C   XDMAX=XDMAX*L
C   WRITE (*,1003)
C   READ  (*,*)      U
C
C   WRITE(*,1100)
C   READ (*,*) TL
C
C   DH  = (IZ-NRDOT)*(MASS-YVDOT)-
C   & (MASS*XG-YRDOT)*(MASS*XG-NVDOT)
C   AA11= ((IZ-NRDOT)*YV-(MASS*XG-YRDOT)*NV)/DH
C   AA12= ((IZ-NRDOT)*(-MASS+YR)-
C   & (MASS*XG-YRDOT)*(-MASS*XG+NR))/DH
C   AA21= ((MASS-YVDOT)*NV-(MASS*XG-NVDOT)*YV)/DH
C   AA22= ((MASS-YVDOT)*(-MASS*XG+NR)-
C   & (MASS*XG-NVDOT)*(-MASS+YR))/DH
C   BB11= ((IZ-NRDOT)*YDRS-(MASS*XG-YRDOT)*NDRS)/DH
C   BB12= ((IZ-NRDOT)*YDRB-(MASS*XG-YRDOT)*NDRB)/DH
C   BB21= ((MASS-YVDOT)*NDRS-(MASS*XG-NVDOT)*YDRS)/DH
C   BB22= ((MASS-YVDOT)*NDRB-(MASS*XG-NVDOT)*YDRB)/DH
C
C   WRITE (*,1004)

```

```

C      READ (*,*)      RATIO
C      BB1=BB11+RATIO*BB12
C      BB2=BB21+RATIO*BB22
C      EPS  =1.D-5
C      ILMAX=1500
C      DO 1 I=1,ITC
        WRITE (*,2001) I,ITC
        TC=TCMIN+(I-1)*(TCMAX-TCMIN)/(ITC-1)
        TCD=TC*L/U
        POLEC=1.0/TCD
        AD1=3.0*POLEC
        AD2=3.0*POLEC**2
        AD3=POLEC**3
        A1=BB1*U*U
        B1=BB2*U*U
        C1=-AD1-(AA11+AA22)*U
        A2=(BB1*AA22-BB2*AA12)*U**3
        B2=(BB2*AA11-BB1*AA21)*U**3
        K1=AD3/((BB2*AA11-BB1*AA21)*U**3)
        C2=AD2-(AA11*AA22-AA12*AA21)*U**2+BB2*U*U*K1
        K2=(C1*B2-C2*B1)/(A1*B2-A2*B1)
        K3=(C2*A1-C1*A2)/(A1*B2-A2*B1)
        DO 2 J=1,IXD
          XD=XDMIN+(J-1)*(XDMAX-XDMIN)/(IXD-1)
C          CALL LINEAR(K1,K2,K3,U,XD,AA11,AA12,AA21,AA22,BB1,BB2,A,TL)
C          CALL RG(4,4,A,WR,WI,0,ZZZ,IV1,FV1,IERR)
C          CALL DSTABL(DEOS,WR,WI,FREQ)
C
          IF (J.GT.1) GO TO 10
          DEOSOO=DEOS
          XDOO  =XD
          LL=0
          GO TO 2
10         DEOSNN=DEOS
          XDNN  =XD
          PR=DEOSNN*DEOSOO
          IF (PR.GT.0.D0) GO TO 3
          LL=LL+1
          IF (LL.GT.3) STOP 1000
          IL=0
          XDO=XDOO
          XDN=XDNN
          DEOSO=DEOSOO
          DEOSN=DEOSNN
6          XDL=XDO
          XDR=XDN
          DEOSL=DEOSO
          DEOSR=DEOSN
          XD=(XDL+XDR)/2.D0
C
          CALL LINEAR(K1,K2,K3,U,XD,AA11,AA12,AA21,AA22,BB1,BB2,A,TL)
C          CALL RG(4,4,A,WR,WI,0,ZZZ,IV1,FV1,IERR)
C          CALL DSTABL(DEOS,WR,WI,FREQ)
C
          DEOSM=DEOS
          XDM=XD

```

```

PRL=DEOSL*DEOSM
PRR=DEOSR*DEOSM
IF (PRL.GT.0.D0) GO TO 5
XDO=XDL
XDN=XDM
DEOSO=DEOSL
DEOSN=DEOSM
IL=IL+1
IF (IL.GT.ILMAX) STOP 3100
DIF=DABS(XDL-XDM)
IF (DIF.GT.EPS) GO TO 6
XD=XDM
GO TO 4
5 IF (PRR.GT.0.D0) STOP 3200
XDO=XDM
XDN=XDR
DEOSO=DEOSM
DEOSN=DEOSR
IL=IL+1
IF (IL.GT.ILMAX) STOP 3100
DIF=DABS(XDM-XDR)
IF (DIF.GT.EPS) GO TO 6
XD=XDM
4 LLL=10+LL
WRITE (LLL,*) XD/L,TC
3 XDOO=XDNN
DEOSOO=DEOSNN
2 CONTINUE
1 CONTINUE
C
1001 FORMAT (' ENTER MIN, MAX, AND INCREMENTS OF TC')
1002 FORMAT (' ENTER MIN, MAX, AND INCREMENTS OF XD')
1003 FORMAT (' ENTER U')
1004 FORMAT (' ENTER BOW/STERN RUDDER RATIO')
1100 FORMAT (' ENTER TIME LAG TL')
2001 FORMAT (2I5)
END
C
SUBROUTINE DSTABL(DEOS,WR,WI,OMEGA)
IMPLICIT DOUBLE PRECISION (A-H,O-Z)
DIMENSION WR(4),WI(4)
DEOS=-1.0D+20
DO 1 I=1,4
IF (WR(I).LT.DEOS) GO TO 1
DEOS=WR(I)
IJ=I
1 CONTINUE
OMEGA=WI(IJ)
OMEGA=DABS(OMEGA)
RETURN
END
C
SUBROUTINE LINEAR(K1,K2,K3,U,XD,AA11,AA12,AA21,AA22,BB1,BB2,A,TL)
IMPLICIT DOUBLE PRECISION (A-H,O-Z)
DOUBLE PRECISION K1,K2,K3
DIMENSION A(4,4)
A(1,1)=0.0D0
A(1,2)=0.0D0
A(1,3)=1.0D0
A(1,4)=0.0D0

```

```

A(2,1)=(BB1*U*U*K1-BB1*U*U*U*K1*TL/XD)/
& (1-BB1*U*U*K1*TL*TL/(2.D0*XD))
A(2,2)=(AA11*U+BB1*U*U*K2-BB1*U*U*K1*TL/XD)/
& (1-BB1*U*U*K1*TL*TL/(2.D0*XD))
A(2,3)=(AA12*U+BB1*U*U*K3+BB1*U*U*U*K1*TL*TL/(2.D0*XD))/
& (1-BB1*U*U*K1*TL*TL/(2.D0*XD))
A(2,4)=(BB1*U*U*K1/XD)/
& (1-BB1*U*U*K1*TL*TL/(2.D0*XD))
A(3,1)=(BB2*U*U*K1-BB2*U*U*U*K1*TL/XD)+
& (BB2*U*U*K1*TL*TL/(2.D0*XD))*
& (BB1*U*U*K1-BB1*U*U*U*K1*TL/XD)/
& (1-BB1*U*U*K1*TL*TL/(2.D0*XD))
A(3,2)=(AA21*U+BB2*U*U*K2-BB2*U*U*K1*TL/XD)+
& (BB2*U*U*K1*TL*TL/(2.D0*XD))*
& (AA11*U+BB1*U*U*K2-BB1*U*U*K1*TL/XD)/
& (1-BB1*U*U*K1*TL*TL/(2.D0*XD))
A(3,3)=(AA22*U+BB2*U*U*K3+BB2*U*U*U*K1*TL*TL/(2.D0*XD))+
& (BB2*U*U*K1*TL*TL/(2.D0*XD))*
& (AA12*U+BB1*U*U*K3+BB1*U*U*U*K1*TL*TL/(2.D0*XD))/
& (1-BB1*U*U*K1*TL*TL/(2.D0*XD))
A(3,4)=(BB2*U*U*K1/XD)+(BB2*U*U*K1*TL*TL/(2.D0*XD))*
& (BB1*U*U*K1/XD)/
& (1-BB1*U*U*K1*TL*TL/(2.D0*XD))
A(4,1)=U
A(4,2)=1.0D0
A(4,3)=0.0D0
A(4,4)=0.0D0
RETURN
END

```


APPENDIX C

```

C   PROGRAM THESIS3.FOR (Time Delay-3rd Order Approx)
C   BIFURCATION ANALYSIS
C
C   IMPLICIT DOUBLE PRECISION (A-H,O-Z)
C   DOUBLE PRECISION K1,K2,K3,L,NR,NV,NDRS,NDRB,IZ,MASS,
C   & NRDOT,NVDOT
C   DIMENSION A(5,5),B(5,5),FV1(5),IV1(5),ZZZ(5,5),ALFR(5),
C   & ALFI(5),BETA(5),WR(5),WI(5)
C
C   OPEN (11,FILE='BIF1.RES',STATUS='NEW')
C   OPEN (12,FILE='BIF2.RES',STATUS='NEW')
C   OPEN (13,FILE='BIF3.RES',STATUS='NEW')
C
C   Vehicle Parameters:
C   WEIGHT=435.0
C   IZ      =45.0
C   L       =7.3
C   RHO     =1.94
C   G       =32.2
C   XG      =0.0104
C   MASS    =WEIGHT/G
C
C   YRDOT  =-0.00178*0.5*RHO*L**4
C   YVDOT  =-0.03430*0.5*RHO*L**3
C   YR     =+0.01187*0.5*RHO*L**3
C   YV     =-0.03896*0.5*RHO*L**2
C   YDRS   =+0.02345*0.5*RHO*L**2
C   YDRB   =+0.02345*0.5*RHO*L**2
C   NRDOT  =-0.00047*0.5*RHO*L**5
C   NVDOT  =-0.00178*0.5*RHO*L**4
C   NR     =-0.01022*0.5*RHO*L**4
C   NV     =-0.00769*0.5*RHO*L**3
C   NDRS   =-0.337*0.02345*0.5*RHO*L**3
C   NDRB   =+0.283*0.02345*0.5*RHO*L**3
C
C   WRITE (*,1001)
C   READ  (*,*)      TCMIN,TCMAX,ITC
C   WRITE (*,1002)
C   READ  (*,*)      XDMIN,XDMAX,IXD
C   XDMIN=XDMIN*L
C   XDMAX=XDMAX*L
C   WRITE (*,1003)
C   READ  (*,*)      U
C   WRITE(*,1100)
C   READ (*,*) TL
C
C   DH  = (IZ-NRDOT)*(MASS-YVDOT)-
C   & (MASS*XG-YRDOT)*(MASS*XG-NVDOT)
C   AA11=((IZ-NRDOT)*YV-(MASS*XG-YRDOT)*NV)/DH
C   AA12=((IZ-NRDOT)*(-MASS+YR)-
C   & (MASS*XG-YRDOT)*(-MASS*XG+NR))/DH
C   AA21=((MASS-YVDOT)*NV-(MASS*XG-NVDOT)*YV)/DH
C   AA22=((MASS-YVDOT)*(-MASS*XG+NR)-
C   & (MASS*XG-NVDOT)*(-MASS+YR))/DH
C   BB11=((IZ-NRDOT)*YDRS-(MASS*XG-YRDOT)*NDRS)/DH
C   BB12=((IZ-NRDOT)*YDRB-(MASS*XG-YRDOT)*NDRB)/DH
C   BB21=((MASS-YVDOT)*NDRS-(MASS*XG-NVDOT)*YDRS)/DH
C   BB22=((MASS-YVDOT)*NDRB-(MASS*XG-NVDOT)*YDRB)/DH
C
C   WRITE (*,1004)

```

```

C      READ (*,*)      RATIO
C      BB1=BB11+RATIO*BB12
C      BB2=BB21+RATIO*BB22
C      EPS  =1.D-5
C      ILMAX=1500
C      DO 1 I=1,ITC
        WRITE (*,2001) I,ITC
        TC=TCMIN+(I-1)*(TCMAX-TCMIN)/(ITC-1)
        TCD=TC*L/U
        POLEC=1.0/TCD
        AD1=3.0*POLEC
        AD2=3.0*POLEC**2
        AD3=POLEC**3
        A1=BB1*U*U
        B1=BB2*U*U
        C1=-AD1-(AA11+AA22)*U
        A2=(BB1*AA22-BB2*AA12)*U**3
        B2=(BB2*AA11-BB1*AA21)*U**3
        K1=AD3/((BB2*AA11-BB1*AA21)*U**3)
        C2=AD2-(AA11*AA22-AA12*AA21)*U**2+BB2*U*U*K1
        K2=(C1*B2-C2*B1)/(A1*B2-A2*B1)
        K3=(C2*A1-C1*A2)/(A1*B2-A2*B1)
        DO 2 J=1,IXD
          XD=XDMIN+(J-1)*(XDMAX-XDMIN)/(IXD-1)
C      CALL LINEAR(K1,K2,K3,U,XD,AA11,AA12,AA21,AA22,BB1,BB2,A,B,TL)
C      CALL RGG(5,5,A,B,ALFR,ALFI,BETA,0,ZZZ,IERR)
C      DO 11 IJE=1,5
C        WR(IJE)=ALFR(IJE)/BETA(IJE)
C        WI(IJE)=ALFI(IJE)/BETA(IJE)
11     CONTINUE
C      CALL DSTABL(DEOS,WR,WI,FREQ)
C      IF (J.GT.1) GO TO 10
C      DEOSOO=DEOS
C      XDOO  =XD
C      LL=0
C      GO TO 2
10     DEOSNN=DEOS
C      XDNN  =XD
C      PR=DEOSNN*DEOSOO
C      IF (PR.GT.0.D0) GO TO 3
C      LL=LL+1
C      IF (LL.GT.3) STOP 1000
C      IL=0
C      XDO=XDOO
C      XDN=XDNN
C      DEOSO=DEOSOO
C      DEOSN=DEOSNN
6      XDL=XDO
C      XDR=XDN
C      DEOSL=DEOSO
C      DEOSR=DEOSN
C      XD=(XDL+XDR)/2.D0
C      CALL LINEAR(K1,K2,K3,U,XD,AA11,AA12,AA21,AA22,BB1,BB2,A,B,TL)
C      CALL RGG(5,5,A,B,ALFR,ALFI,BETA,0,ZZZ,IERR)

```

```

DO 12 IJE=1,5
  WR(IJE)=ALFR(IJE)/BETA(IJE)
  WI(IJE)=ALFI(IJE)/BETA(IJE)
12 CONTINUE
CALL DSTABL(DEOS,WR,WI,FREQ)

C
DEOSM=DEOS
XDM=XD
PRL=DEOSL*DEOSM
PRR=DEOSR*DEOSM
IF (PRL.GT.0.D0) GO TO 5
XDO=XDL
XDN=XDM
DEOSO=DEOSL
DEOSN=DEOSM
IL=IL+1
IF (IL.GT.ILMAX) STOP 3100
DIF=DABS(XDL-XDM)
IF (DIF.GT.EPS) GO TO 6
XD=XDM
GO TO 4
5 IF (PRR.GT.0.D0) STOP 3200
XDO=XDM
XDN=XDR
DEOSO=DEOSM
DEOSN=DEOSR
IL=IL+1
IF (IL.GT.ILMAX) STOP 3100
DIF=DABS(XDM-XDR)
IF (DIF.GT.EPS) GO TO 6
XD=XDM
4 LLL=10+LL
WRITE (LLL,*) XD/L,TC
3 XDOO=XDNN
DEOSOO=DEOSNN
2 CONTINUE
1 CONTINUE

C
1001 FORMAT (' ENTER MIN, MAX, AND INCREMENTS OF TC')
1002 FORMAT (' ENTER MIN, MAX, AND INCREMENTS OF XD')
1003 FORMAT (' ENTER U')
1004 FORMAT (' ENTER BOW/STERN RUDDER RATIO')
1100 FORMAT (' ENTER TIME LAG TL')
2001 FORMAT (2I5)
END

C
SUBROUTINE DSTABL(DEOS,WR,WI,OMEGA)
IMPLICIT DOUBLE PRECISION (A-H,O-Z)
DIMENSION WR(5),WI(5)
DEOS=-1.0D+20
DO 1 I=1,5
  IF (WR(I).LT.DEOS) GO TO 1
  DEOS=WR(I)
  IJ=I
1 CONTINUE
OMEGA=WI(IJ)
OMEGA=DABS(OMEGA)
RETURN
END

C

```

```

SUBROUTINE LINEAR(K1,K2,K3,U,XD,AA11,AA12,AA21,AA22,BB1,BB2,
  A,B,TL)
  IMPLICIT DOUBLE PRECISION (A-H,O-Z)
  DOUBLE PRECISION K1,K2,K3
  DIMENSION A(5,5),B(5,5)
  A(1,1)=0.0D0
  A(1,2)=0.0D0
  A(1,3)=1.0D0
  A(1,4)=0.0D0
  A(1,5)=0.0D0
  A(2,1)=0.0D0
  A(2,2)=0.0D0
  A(2,3)=0.0D0
  A(2,4)=0.0D0
  A(2,5)=1.0D0
  A(3,1)=(BB1*U*U*K1-BB1*U*U*U*K1*TL/XD)
  A(3,2)=(AA11*U+BB1*U*U*K2-BB1*U*U*K1*TL/XD)
  A(3,3)=(AA12*U+BB1*U*U*K3+BB1*U*U*U*K1*TL*TL/(2.D0*XD))
  A(3,4)=(BB1*U*U*K1/XD)
  A(3,5)=(-1.D0+BB1*U*U*K1*TL*TL/(2.D0*XD))
  A(4,1)=U
  A(4,2)=1.0D0
  A(4,3)=0.0D0
  A(4,4)=0.0D0
  A(4,5)=0.0D0
  A(5,1)=(BB2*U*U*K1-BB2*U*U*U*K1*TL/XD)
  A(5,2)=(AA21*U+BB2*U*U*K2-BB2*U*U*K1*TL/XD)
  A(5,3)=(AA22*U+BB2*U*U*K3+BB2*U*U*U*K1*TL*TL/(2.D0*XD))
  A(5,4)=(BB2*U*U*K1/XD)
  A(5,5)=(BB2*U*U*U*K1*TL*TL/(2.D0*XD))

C
  B(1,1)=1.0D0
  B(1,2)=0.0D0
  B(1,3)=0.0D0
  B(1,4)=0.0D0
  B(1,5)=0.0D0
  B(2,1)=0.0D0
  B(2,2)=1.0D0
  B(2,3)=0.0D0
  B(2,4)=0.0D0
  B(2,5)=0.0D0
  B(3,1)=0.0D0
  B(3,2)=0.0D0
  B(3,3)=(BB1*U*U*U*K1*TL*TL*TL/(6.D0*XD))
  B(3,4)=0.0D0
  B(3,5)=(BB1*U*U*K1*TL*TL*TL/(6.D0*XD))
  B(4,1)=0.0D0
  B(4,2)=0.0D0
  B(4,3)=0.0D0
  B(4,4)=1.0D0
  B(4,5)=0.0D0
  B(5,1)=0.0D0
  B(5,2)=0.0D0
  B(5,3)=(1.0D0+BB2*U*U*U*K1*TL*TL*TL/(6.D0*XD))
  B(5,4)=0.0D0
  B(5,5)=(BB2*U*U*K1*TL*TL*TL/(6.D0*XD))
  RETURN
  END

```

APPENDIX D

```

C      PROGRAM THESISN.FOR (Time Delay - Newton's Iteration Method)
C
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      DOUBLE PRECISION K1,K2,K3,L,NR,NV,NDRS,NDRB,IZ,MASS,
& NRDOT,NVDOT
      DIMENSION A(4,4),FV1(4),IV1(4),ZZZ(4,4),WR(4),WI(4)
C
      OPEN (11,FILE='BIF1.RES',STATUS='NEW')
C
C      Vehicle Parameters:
      WEIGHT=435.0
      IZ      =45.0
      L       =7.3
      RHO     =1.94
      G       =32.2
      XG      =0.0104
      MASS    =WEIGHT/G
C
      YRDOT =-0.00178*0.5*RHO*L**4
      YVDOT =-0.03430*0.5*RHO*L**3
      YR     =+0.01187*0.5*RHO*L**3
      YV     =-0.03896*0.5*RHO*L**2
      YDRS   =+0.02345*0.5*RHO*L**2
      YDRB   =+0.02345*0.5*RHO*L**2
      NRDOT  =-0.00047*0.5*RHO*L**5
      NVDOT  =-0.00178*0.5*RHO*L**4
      NR     = 0.01022*0.5*RHO*L**4
      NV     =-0.00769*0.5*RHO*L**3
      NDRS   =-0.337*0.02345*0.5*RHO*L**3
      NDRB   =+0.283*0.02345*0.5*RHO*L**3
C
      WRITE (*,1001)
      READ  (*,*)      TCMIN,TCMAX,ITC
      WRITE (*,1003)
      READ  (*,*)      U
C
      DH = (IZ-NRDOT)*(MASS-YVDOT)-
& (MASS*XG-YRDOT)*(MASS*XG-NVDOT)
      AA11=((IZ-NRDOT)*YV-(MASS*XG-YRDOT)*NV)/DH
      AA12=((IZ-NRDOT)*(-MASS+YR)-
& (MASS*XG-YRDOT)*(-MASS*XG+NR))/DH
      AA21=((MASS-YVDOT)*NV-(MASS*XG-NVDOT)*YV)/DH
      AA22=((MASS-YVDOT)*(-MASS*XG+NR)-
& (MASS*XG-NVDOT)*(-MASS+YR))/DH
      BB11=((IZ-NRDOT)*YDRS-(MASS*XG-YRDOT)*NDRS)/DH
      BB12=((IZ-NRDOT)*YDRB-(MASS*XG-YRDOT)*NDRB)/DH
      BB21=((MASS-YVDOT)*NDRS-(MASS*XG-NVDOT)*YDRS)/DH
      BB22=((MASS-YVDOT)*NDRB-(MASS*XG-NVDOT)*YDRB)/DH
C
      WRITE (*,1004)
      READ  (*,*)      RATIO
C
      BB1=BB11+RATIO*BB12
      BB2=BB21+RATIO*BB22
C
      WRITE (*,1005)
      READ  (*,*)      TL
C
      EPS=1.D-10
      ITMAX=1000

```

```

C
DO 1 I=1,ITC
  WRITE (*,2001) I,ITC
  TC=TCMIN+(I-1)*(TCMAX-TCMIN)/(ITC-1)
  TCD=TC*L/U
  POLEC=1.0/TCD
  AD1=3.0*POLEC
  AD2=3.0*POLEC**2
  AD3=POLEC**3
  A1=BB1*U*U
  B1=BB2*U*U
  C1=-AD1-(AA11+AA22)*U
  A2=(BB1*AA22-BB2*AA12)*U**3
  B2=(BB2*AA11-BB1*AA21)*U**3
  K1=AD3/((BB2*AA11-BB1*AA21)*U**3)
  C2=AD2-(AA11*AA22-AA12*AA21)*U**2+BB2*U*U*K1
  K2=(C1*B2-C2*B1)/(A1*B2-A2*B1)
  K3=(C2*A1-C1*A2)/(A1*B2-A2*B1)

C
  A1= (AA11*BB2-AA21*BB1)*U*U*U*K1
  A2= (AA11*AA22-AA12*AA21)*U*U+(AA11*BB2-AA21*BB1)*U*U*U*K3+
&   (AA22*BB1-AA12*BB2)*U*U*U*K2-BB2*U*U*K1
  A3=-(AA11+AA22)*U-(BB1*K2+BB2*K3)*U*U
  B0= (AA11*BB2-AA21*BB1)*U*U*U*U*K1
  B1=-(AA12*BB2-AA22*BB1)*U*U*U*K1-BB2*U*U*U*K1
  B2=-BB1*U*U*K1

C
C
C
  COMPUTE INITIAL APPROXIMATION FOR OMEGA (TL=0)

  AQ=B2*A3-B1
  BQ=B1*A2-B2*A1-B0*A3
  CQ=B0*A1
  DET=BQ**2-4.0D0*AQ*CQ
  IF (DET.LT.0.D0) GO TO 1
  ZT1=(-BQ+DSQRT(DET))/(2.0D0*AQ)
  ZT2=(-BQ-DSQRT(DET))/(2.0D0*AQ)
  IF (ZT1.GE.0.D0) OM=DSQRT(ZT1)
  IF (ZT2.GE.0.D0) OM=DSQRT(ZT2)
  ALPHA1=AQ
  ALPHA2=BQ
  ALPHA3=CQ
  BETA1=-B2
  BETA2=B0+B2*A2-B1*A3
  BETA3=B1*A1-B0*A2

C
C
C
  COMPUTE EXACT OMEGA USING NEWTON'S ITERATION

  OMOLD=OM
3  F=(BETA1*OMOLD**5+BETA2*OMOLD**3+BETA3*OMOLD)*DSIN(OMOLD*TL)
&   +(ALPHA1*OMOLD**4+ALPHA2*OMOLD**2+ALPHA3)*DCOS(OMOLD*TL)
  FPRIME=(5.0D0*BETA1*OMOLD**4+3.0D0*BETA2*OMOLD**2+BETA3)
&   *DSIN(OMOLD*TL)+(BETA1*OMOLD**5+BETA2*OMOLD**3+BETA3*OMOLD)
&   *TL*DCOS(OMOLD*TL)+(4.0D0*ALPHA1*OMOLD**3+2.0D0*ALPHA2*OMOLD)
&   *DCOS(OMOLD*TL)-(ALPHA1*OMOLD**4+ALPHA2*OMOLD**2+ALPHA3)
&   *TL*DSIN(OMOLD*TL)
  IF (FPRIME.EQ.0.D0) STOP 1112
  IF (F.EQ.0.D0) GO TO 2
  OMNEW=OMOLD-F/FPRIME
  OMDIF=DABS(OMNEW-OMOLD)
  IF (OMDIF.LE.EPS) GO TO 2

```

```

      OMOLD=OMNEW
      IT=IT+1
      IF (IT.GT.ITMAX) STOP 1111
      GO TO 3
C
      2  OM=OMNEW
        XDNUM=DSQRT((B0-B2*OM**2)**2+(B1*OM)**2)
        XDDEN=OM*DSQRT((A1-A3*OM**2)**2+(A2*OM-OM**3)**2)
        XD=XDNUM/XDDEN
C
        WRITE(11,*) XD,TC,OM
C
      1  CONTINUE

1001 FORMAT (' ENTER MIN, MAX, AND INCREMENTS OF TC')
1003 FORMAT (' ENTER U')
1004 FORMAT (' ENTER BOW/STERN RUDDER RATIO')
1005 FORMAT (' ENTER TIME LAG')
2001 FORMAT (2I5)
      END

```

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